Investigation of the Effect of Maximum Sideband Level Inequality to the Solution Time in Time-Modulated Small Sized Linear Arrays

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Abstract

In this study, the effect of the maximum sideband inequality to the solution time which is proposed to suppression of the maximum sideband level caused by switching in time-modulated linear arrays is investigated. The relation between the change of element number and the solution time is interpreted by comparing the inequality and conventional method for equispaced linear arrays.

Key words: Time-modulation, linear arrays, differential evolution algorithm, sideband suppression.

1. Introduction

After the introduction of time modulation concept [1], it has been firstly used to synthesize low/ultra-low sidelobe patterns with low dynamic range ratio (DRR) through the time parameter [2]. While this concept may provide the desired patterns with lower DRR, also brings along some difficulties and the main problem of this technique is the existence of the sidebands caused by periodical switching. As a remedy to this difficulty, the suppression of sidebands which may be considered as a power loss or may be even interfere with useful signals at harmonic frequencies is provided using several switching sequences such as variable aperture size (VAS) [3], pulse shift [4] and pulse split [5,6] via different optimizers such as differential evolution algorithm (DE) [7,8] and particle swarm optimization (PSO) [9].

All these studies have been carried out via a technique which may be named as the conventional method and may be described as the sampling of the harmonic frequencies with certain sensitivity and reducing the maximum valued one among the sample space [3-9]. Considering that there is infinite number of harmonics, one major disadvantage of this process is the determination of which of the sidebands to be sampled and the uncertainty in the sampling sensitivity. Despite this uncertainty, this process is usually performed with the assumption of “the first sideband is bigger than all other harmonics”. As an alternative to this approach, it is asserted that there may be an upper bound of these sidebands and with the "all side bands should remain below the value of this inequality" idea, both the processing time may be shortened and the existing uncertainties in the conventional method may be removed [10-11].

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How the inequality affects the solution time under variable element number has not been analyzed and the purpose of this study is to examine this effect. In this study, the solution times of the conventional method and the inequality have been examined and interpreted over a reference scenario with variable element equispaced linear array.

The study consists of four sections and is organized as follows: In the second chapter a brief background about the concept of time modulation has been given. In the third section, two different methods that are used for sideband suppression have been introduced, comparisons in terms of solution times have been made and comments on the comparisons have been provided. Finally, in the fourth section the study has been concluded.

2. A Brief Theory of Time-Modulation

The time modulation is the periodically switching the desired elements of a conventional antenna array. The block diagram of a potential receiver for this type structure is given in Figure 1. If an N element linear array whose elements are placed on z-axis is considered, under far-field approximation the array factor for this array may be written as:

$$F(\theta) = \sum_{n=0}^{N-1} W_n \exp\{jkz_n \cos\theta\} \tag{1}$$

where W_n and z_n represents the complex amplitude excitations and the coordinate of the nth element, respectively. Additionally, θ represents the elevation angle in measured from the z-axis and $k = 2\pi/\lambda$ represents the wavenumber at central operating frequency. If each element of this array is switched by some high speed RF switches is considered, under $w_0/w_p \gg 1$ approximation where w_0 being the central operating frequency and w_p being the switching frequency, the array factor becomes the form of:

$$F(\theta) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} W_n \exp\{jkz_n \cos\theta\}. \tag{2}$$

Here, $m = 0$ term represents the central operation band and $|m| \geq 1$ terms are called the sidebands. Additionally, a periodic function $U_n\ (t)$ with a period of T_p may be written in terms of complex Fourier series as:

$$U_n(t) = \sum_{m=-\infty}^{\infty} C_n^m \exp\{jw_P mt\} \tag{3}$$

where C_n^m represents the complex Fourier coefficients of time dependent terms caused by periodic switching and defined as:

$$C_n^m = \frac{1}{T_P} \int_{t_0}^{t_0+T_P} U_n(t) \exp\{-jw_P mt\}. \tag{4}$$

Under VAS switching scheme C_n^m term may be written as $C_n^0 = \tau_n/T_P$ where τ_n being the total switch-on duration of corresponding element for main operation frequency, and it may be written for sidebands as:

$$C_n^m = \frac{\sin(\pi m\, \tau_n/T_P)}{\pi m} \exp\{-j\pi m\, \tau_n/T_P\}. \tag{5}$$

Hence, the array factor for a time modulated array may be written as:

$$F(\theta) = \begin{cases} \sum_n \frac{W_n \tau_n}{T_P} \exp\{jkz_n \cos\theta\} & , m = 0 \\ \sum_n \frac{W_n \sin(\pi m\, \tau_n/T_P)}{\pi m} \exp\{jkz_n \cos\theta - j\pi m\, \tau_n/T_P\} & , |m| \geq 1 \end{cases} \tag{6}$$

In this expression, the usage of time as a design parameter is obvious. Here, it must be noted that Equation (6) only remains valid for a linear array switched by VAS time scheme under far-field and $\mathrm{w_0/w_p} \gg 1$ approximations.

3. Analysis, Results and Discussion

The flow charts illustrating the usage of conventional method and the inequality used for the suppression of sidebands caused by time-modulation are given in Figure 2 and Figure 3, respectively. In conventional method, the sideband level is calculated on the principle of sampling the three dimensional space with a certain precision, repeating the summation of the fields of each of the array elements at these space samples for desired number of harmonics and eventually the selection of amplitude of the sample point having the highest field value the among the sample space. The general belief for this method is the sideband calculations performed in the first harmonic are sufficient to obtain maximum level and this general belief is also used in this study. The inequality method is based on the idea of all formed sidebands should remain below the threshold value calculated via the inequality and this inequality is defined as:

$$\Psi_0 \leq \frac{\sum_n I_n \sin(\pi\tau_n)}{\pi \sum_n \xi_n} \tag{7}$$

where Ψ_0 being the normalized sideband level. In this expression, I_n represents the normalized excitation amplitudes, τ_n represents the normalized total switch-on durations and ξ_n is called the dynamic excitation which is defined as $\xi_n = I_n\tau_n$.

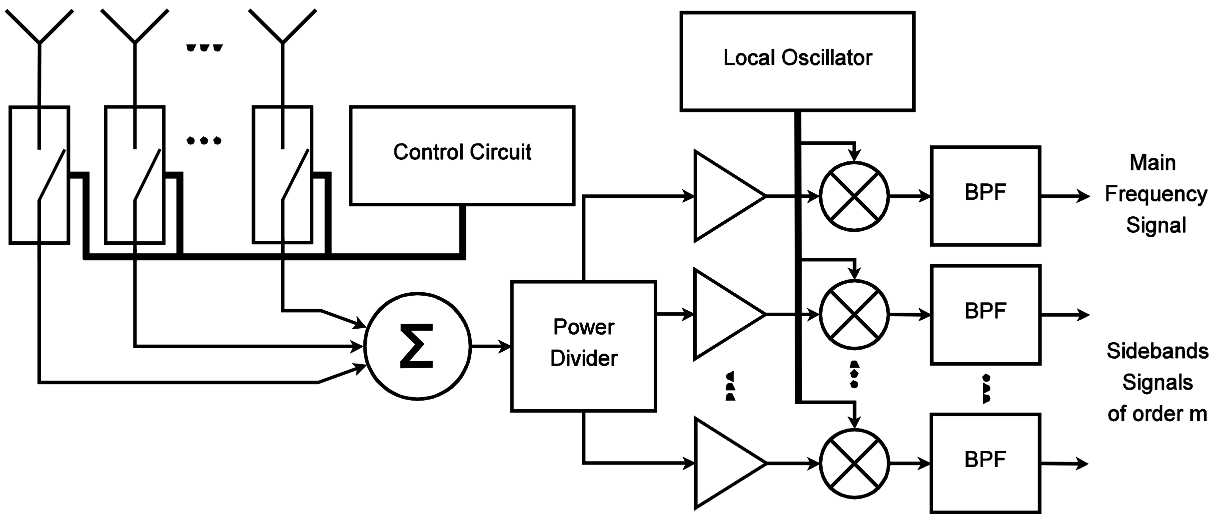

Figure 1. Block diagram of potential receiver structure for a time-modulated linear array

Since both methods are used for sideband suppression, in order to make a fair solution time comparison, the sideband suppression problem of an *N* element small sized linear array has been selected as the reference scenario. In this relatively simple scenario, all infinite sidebands are intended to be suppressed below -30 dB under variable element number *N*. For this purpose, a zero phased linear array consists of isotropic sources with the interelement spacing of $0.5\lambda_0$ placed along *z*-axis has been considered and DE algorithm with “DE/best/1/bin” mutation and crossover strategy has been selected as the as the optimization tool. For this scenario, the mutation factor, the crossover factor and population size has been selected as *F*=0.6, *Cr*=0.95, *P*=40, respectively, as the algorithm parameters. The algorithm has been set to be terminated when the cost functions f_c and f_i which are defined as:

$$f_c = \max_{m=1}\{\Lambda_c\} \tag{8}$$

and

$$f_i = \Lambda_i \tag{9}$$

where Λ_c and Λ_i represents the sideband level for the conventional solution and inequality value, respectively, are reduced below the desired value. Since the solution time comparison is wanted to be conducted over a small sized array, the element number has been set to be vary between 10 and 50 (i.e. $\mathrm{N} \in [10{,}50]$). All calculations are performed on a standard commercial PC having Intel i5-2400 CPU@3.10 GHz with 3.16 GB of RAM.

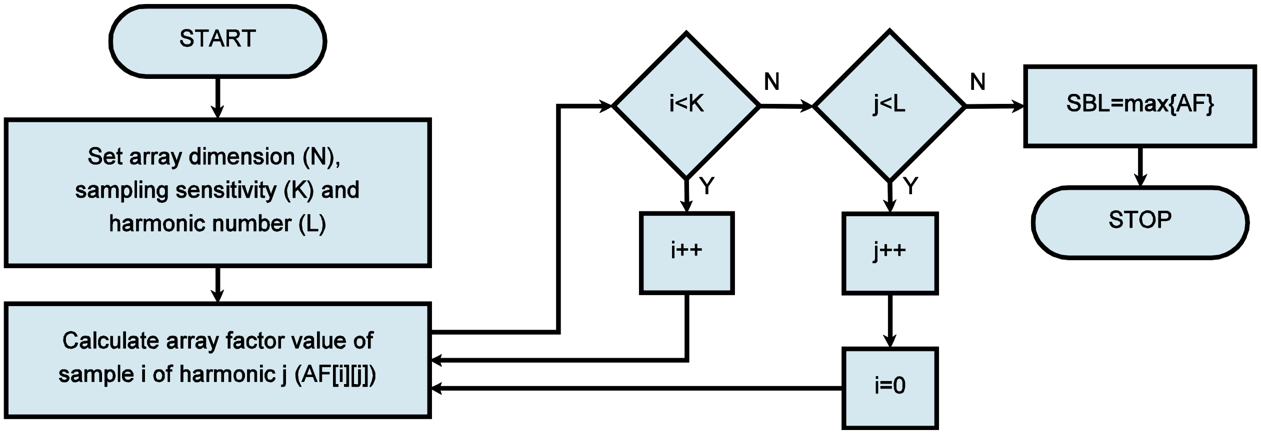

Figure 2. Flow chart of maximum sideband level calculation for a single harmonic frequency with conventional method

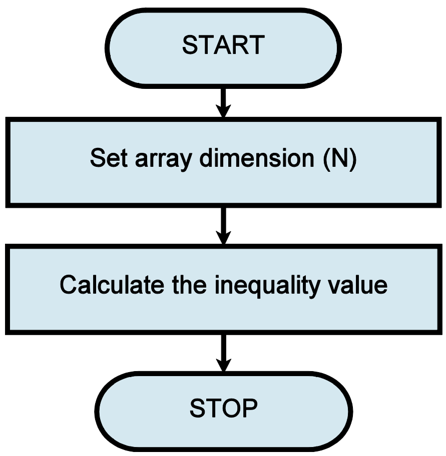

Figure 3. Flow chart of maximum sideband level calculation using inequality

Since the methods are running on a random-based optimization tool, the obtained singular solutions may cause deceptive consequences. Therefore, instead of making a comparison with the best individual solutions, an observation has been made on the basis of the mean of K independent experiments. The element number-Solution time graph of the comparison conducted based on the solution time over 40 independent experiments is presented in Figure 4 and this figure constitutes an illustrative example in terms of solution time for both methods. According to Figure 4, while the process conducted via inequality has been finished 0.0633 seconds in average for 10 elements which is the lower bound of the element number, this value has been observed as 4.0664 seconds for conventional method. These values have been found as 5.0516 and 161.6109 seconds for the inequality and the conventional method, respectively, for 50 elements which is the upper bound of the element number definition range. The shortest and the longest solution times calculated by using inequality for the lower and upper element number limits are observed as; 0.046 seconds at least, 0.203 seconds at most for the lower bound, 3.141 seconds at least and 7.031 seconds at most for the upper bound. These values are calculated via conventional methods as 2.656 seconds at least, 6.125 seconds at most for lower bound, 89.156

seconds at least, 261.11 seconds at most for the upper bound. These values clearly show that the inequality exhibits highly successful results in terms of solution time compared to the traditional method. Furthermore, the calculated deviations for the solution times are also indicated in Figure 4.

As it can be seen from Figure 4, the solution time for conventional method tends to an exponential increase. Considering Figure 4 again, although the optimization tool and methods used have a non-linear characteristic though, it can be interpreted that the usage of the inequality makes the solution times closer to linear compared to conventional method. For lower element number bound, the deviation in solution times is in a negligible level for both methods. However, with the increase in the number of elements, the deviation rate for the conventional method becomes unstable compared to the inequality. This situation leads to the conclusion that more predictable and stable solution times can be provided; together with the result of usage of the inequality shortens the process time.

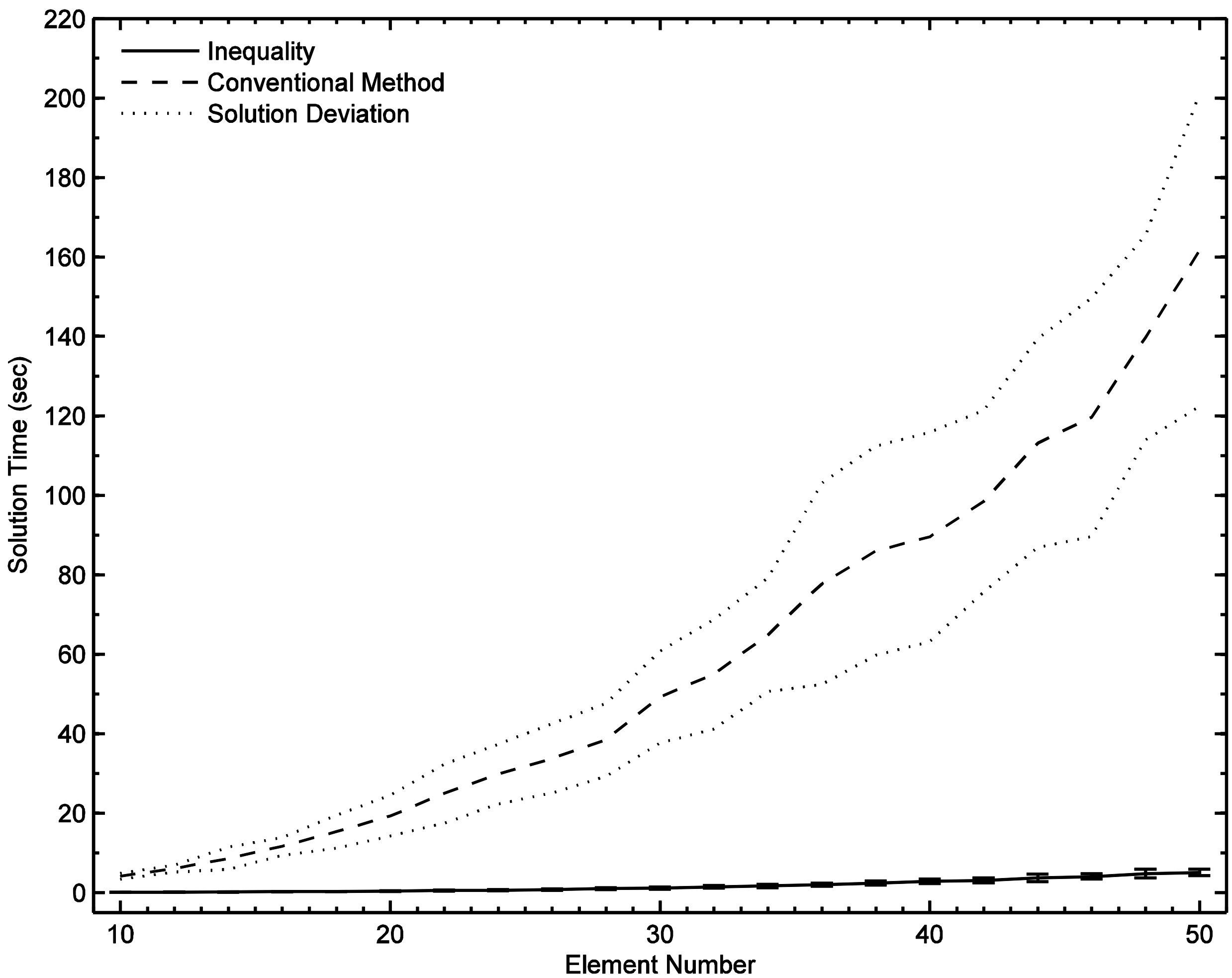

Figure 4. Element number-Solution time variation graph for the conventional method and the inequality (all values have been calculated via DE)

Conclusions

In conclusion, in this study, an analysis of two different methods that may be used for sideband suppression in time modulated arrays has been performed over variable element numbered scenarios of small sized arrays in terms of solution time. The vast superiority in terms of solution time of inequality method on sideband sampling technique which can be called the classical or conventional method has been shown over the reference scenario using a small sized linear array. Although both methods exhibit a non-linear solution time characteristic depending on the "element number" caused by the structure of the algorithm as well as the array factor, it is shown that the inequality method relatively brings the solution times closer to linear with respect to conventional technique. Also it is shown that usage of the inequality provides more stable solution time trend than the conventional method.

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