Wind Farm Optimization

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Abstract

The main goal of this paper is to use a mixed integer linear program to formulate the optimization process of a wind farm. As a start point, a grid was superimposed into the wind farm, in which grid points represent possible wind turbine location. During the optimization process, proximity and wind interference between wind turbines were considered in order to found the power loss of the wind farm. Power loss was found by using wind interference coefficient, which is a function of wind intensity interference factor (WIIF), weibull distribution and output power of the wind turbines. Two different programs; Genetic Algorithm and Lingo, were used to solve the MILP optimization formula and results were compared in the conclusion part.

Keywords: Micro-siting, Wind Farm, Wake effect, Genetic Algorithm, LINGO, Maximum power

1. Introduction

Nowadays, one of the most challenging topics among the different kinds of renewable energy is the wind energy. It seems that humans realized the benefit of the wind in 200 B.C., when Chinese first invented a windmill. Wind is used in a process to generate useful kind of energies like mechanical and electrical energies. Wind energy or wind power refers to this conversion process in which wind turbines are used. Wind power is an alternative energy types to fossil fuel and has no negative effects on nature like fossils. Since wind is a sustainable energy source, wind energy has become widespread during last 20 years. Although wind energy has several disadvantages, it seems that the positive sides of this renewable energy are more attractive for worldwide industry. As wind turbines have became common during years, people started to think about using multiple wind turbines in a same location in order to gain more power. This idea brought about concept of wind farm which consists of many individual wind turbines, up to over a hundred in some cases, in the same place to produce electric power. Most of the largest wind farms are located in United States and China. For instance, Alta Energy Centre with 1020 MW capacity in USA and Dabancheng wind farm with 500 MW in China are two examples of these wind farms [1]. The number and the type of wind turbines may vary from project to project. But the main goal in all the wind farms is to maximize the captured energy while minimizing the overall cost. Therefore, all the necessary conditions must be analyzed during the construction of the wind farm to make it more efficient.

As it was mentioned before, the main objective of a wind farm construction is to capture more energy by placing wind turbines in a place that they can produce more power in comparison to other positions. Micro-siting indicates the exact position of each wind turbine in a wind farm since the place of the wind turbine matters in a wind farm. The exact position of wind turbines can be found by using different methods such as Finite Difference methods and Genetic Algorithm. No matter what the method is, there are some factors that should be analyzed during

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the optimization process. Proximity and interferences between wind turbines are the most two important criteria in the micro-siting. The present research work has been carried out to find the best locations for wind turbines in a hypothetical wind farm in order to produce maximum power. The study has the following objectives:

1. Use an optimization model in order to minimize the power loss and maximize the captured power
2. Solve the optimization model by using Genetic Algorithm and Lingo
3. Compare the results for genetic algorithm and lingo for different wind farm models

There are varieties of optimization methods to solve the maximization issue in a wind farm. However, in the present work a mixed integer linear problem (MILP) [2] was used to analysis the proximity and interference among wind turbines. A grid is imposed onto the wind farm and each grid point shows the possible location of a wind turbine. The Proximity and interference between wind turbines were defined by the edges of the grid. Lingo 13.0 and Matlab (Genetic Algorithm) programs were used to solve the MILP model for different dimensions.

2. Method

2.1. Wind Turbine Wake

When the wind passes through the rotor of the wind turbine, a reduction in speed occurs behind the rotor. It means the downwind air of the wind turbine has a lower wind speed and higher turbulence, therefore, for downstream wind turbine energy production is not the same with the upstream wind turbine since there is a deficit in wind speed [3]. Wake effects refer to this wind speed reduction and diminish in energy production in a wind farm based on interactions between wind turbines [3]. A wake can be divided into two regions which are near and far wake regions. The near area is the area just behind the wind turbine rotor and is approximately about one rotor diameter. Near wake region is related to rotor's properties such as number of blades, blade aerodynamics, including stalled flow, 3-D effects and the tip vortices [4]. The far wake region, on the other hand, is the area after the near wake region and is more important in wind farm analysis. In this region the main focus is on wake models, wake interference, turbulence models, environmental and atmospheric conditions and topographical effects [4]. Therefore, while the near wake region is based on characteristics of wind turbine, the far wake region is related to the interaction among wind turbines in a wind farm.

2.2. Velocity Deficit

As it was mentioned before, the velocity of the downstream turbine and upstream turbine is not the same and has to be calculated. It matters that how many wind turbines are placed in upstream and have effect over a downwind turbine. There are different methods to modelling the wake effect among wind turbines such as Ainslie's model [5], Frandsen's model [6], the Mosaic Tile model [7] and Jensen's model [8]. In this work, the N.O. Jensen wake model was used. It is a simple single wake model based on the assumption that a wake expanding linearly and the wind speed has a rectangular profile [9]. This model also called modified PARK model (GH, 2006 and Jensen, 1983) which describes the turbine wake and defines a velocity model for downwind turbine when there is no other turbines between downwind and upwind turbines. When there are more than two wind turbines in a wind farm, it is quite possible that one turbine is affected by several wakes. Therefore, all the effects should be taken into consider. By using merged wake model, multiple wakes effect over a single downwind turbine is calculated. The main assumption here is that the kinetic energy deficit of interacting wakes is equal to the sum of the energy deficits of the individual wakes [3]. The Figure 1 indicates that wind turbine 'j' or downwind wind turbine is not only affected by upwind turbine 'i', it is also in the wake direction of other upwind turbines.

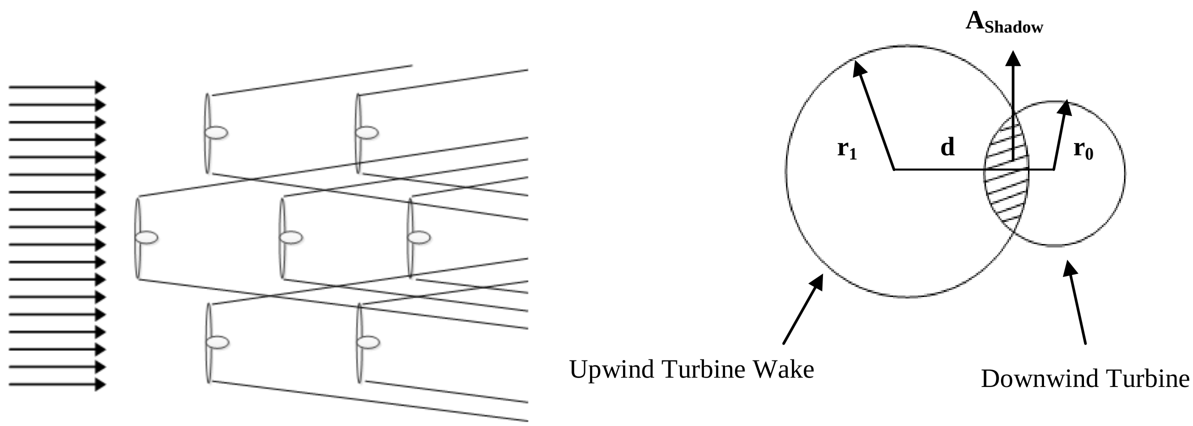

Figure 1. Illustration of Multiple Wakes and A_{shadow} of Downwind Turbine in a Wind Farm

Therefore, in these situations the following formula is used, where C_t is the thrust coefficient and A_0 is the downwind turbine's area.

$$V_{downwind} = V_{upwind}\left[1 - \sum_{i=1}^{n}(1 - \sqrt{1 - C_t})\left(\frac{r_0}{r_1}\right)^2 \left(\frac{A_{shadow}}{A_0}\right)\right] \tag{1}$$

A_{Shadow} is calculated from the following formula;

$$A_{Shadow} = arccos\left(\frac{r_1^2+d^2-r_0^2}{2*r_1*d}\right) * r_1^2 + \arccos\left(\frac{r_0^2+d^2-r_1^2}{2*r_0*d}\right) * r_2^2 - sin\left[\arccos\left(\frac{r_1^2+d^2-r_0^2}{2*r_1*d}\right)\right] * r_1 * d \tag{2}$$

2.3. Proximity Constraint

Proximity refers to distance between wind turbines in a wind farm. It is important to define the distance between wind turbines before start to model the optimization formula. The energy production is affected by the distance between upwind turbines and downwind turbines and it is decreasing as the space among wind turbines reduces. This happens because one of the main factors in increasing the interactions and wake effects in a wind farm is the proximity between turbines. Therefore, by decreasing the distance between wind turbines, the wake effects increase. Moreover, if turbines placed too close to each other, blades may crash with one another. The proximity introduces a limit for minimum distance for vertices in a grid to guarantee that there will not be a physical clash between turbine blades. Therefore, no other turbines can be located in another turbine proximity zone. It was mentioned that the model is developed by superimposing an orthogonal grid onto the physical topography with regular lattice spacing as a function of wind turbine diameter, in which vertices represents the wind turbines. If a turbine placed on the solid vertex, no other turbine can be positioned in around vertices because the around vertices are too close to the main wind turbine and may be overlapping each other proximity zone. Therefore, these positions are not suitable to locate another turbine. To model this proximity constraint we can assume two vertices i and j that have an edge with each other. If a turbine positioned in i vertex, the around space cannot occupied by vertex j. It can be written in following form [2]:

$$x_i + x_j \leq 1 , \forall (i,j) \in E \tag{3}$$

Here E represents the union of the interference and proximity edges. This constraint occurs for every vertex and edge pair in a grid. However, this formulation can be improved by expanding it for a larger subset of vertices accordance with their proximity zone. This larger subset of vertices is defined as a maximal clique Q which is consists of all vertices that are connected to each other by an edge [10,11].

2.4. Interference Constraint

In this work, the interference model is not based on an arbitrary distance between vertices in a grid. However, it is related to total captured power. That means a vertex may not only have edges with closest vertices, but it may also have edges with other vertices that are not in its proximity zone. If x_u represents a turbine positioned at vertex u and x_v represents a turbine positioned at v, then I_{uv} refers to power loss because of the interference between u and v vertices. In other words, I_{uv} is wake interference coefficient and it is based on wake intensity interference factor and is discussed in following parts. The mathematical formulation for this constraint is then [10,11];

$$I_{ij}(x_i + x_j - 1) \leq Z_i \tag{4}$$

Here Z_i is zero if there is not any turbine at x_i and x_j. And if Z_i=1, then a turbine is positioned at both x_i and x_j and the gain power reduces by amount of I_{ij}.

2.5. Wake Intensity Interference Factor

The next step in formulation process is to calculate the I_{ij} [10, 11]. The main factor in I_{ij} calculation is to analysis the intensity of the interference between wind turbines, which is related to geometry of the wind turbines and wind direction distribution. Therefore, a wake intensity interference factor (WIIF) for various positions is introduced in this section.

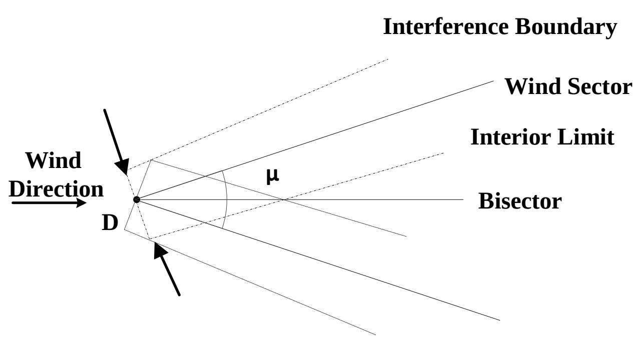

Figure 2. Turbine Wake Depiction

Any particular wind direction is assumed to be equi-probable in every wind sector. It means, an expected wind direction exists for each sector and it is called the bisector of the wind sector. Moreover each sector has an angle of $\mu = \frac{2\pi}{n}$ radians where n is the number of sectors. Since wind direction varies through all possible angles within the wind sectors, the extreme angles of the turbine wake represents the interference boundary and interior limit of that wind sector. Any point in the interference boundary is affected by the turbine wake for that wind direction. However, it should be noted that any particular wind direction can be considered here, therefore, there are a range of wind direction angles where the turbine wake affected that point. Note that, turbine wake is expanding by amount of α angle and therefore interference boundary, interior limit and wind sector are not parallel to each other. The wind interference intensity factor (WIIF) is defined as a ratio of this angle to μ and denotes by $\Psi(r,\theta)$ and varies from 0 to 1.

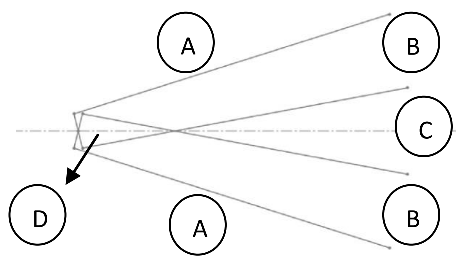

Figure 3. WIIF for Four Region

To find the exact value of the WIIF, it is divided in to four distinct regions which should be calculated separately. These regions are shown in the Figure 3. In the first region (A), WIIF is zero when the point is outside of the interference boundary ($\Psi(r,\theta)_A = 0$); however, for remain three regions it should be calculated separately. The second region (B) is somewhere between the interference boundary and the interior limit. The third region (C) is where the downwind turbine is placed in the interior limit at a radius greater than r_B [10,11]. The last region (D) represents a case when downwind turbine is located within the interior boundary at a radius less than r_B. It means that downwind turbine is too close to upwind turbine that the WIIF takes its maximum value, 1. To summarize four distinct regions WIIF is defined below.

$$\Psi(r,\theta) = \begin{cases} \frac{1}{\mu}\left(\frac{\mu}{2} + \xi(r) - |\theta|\right) & |\varphi(r)| < |\theta(r)| < |\omega(r)| \quad r \geq \frac{D}{r} \\ \frac{2*\xi(r)}{\mu} & |\theta(r)| < |\varphi(r)| \quad r \geq r_B \\ 1 & |\theta(r)| < |\varphi(r)| \quad \frac{D}{2} \leq r \leq r_B \\ 0 & otherwise \end{cases} \tag{5}$$

Where;

$$\xi(r) = \alpha + \arcsin\left(\frac{D*Cos\alpha}{2r}\right) \tag{6}$$

$$\varphi(r) = \pm\left(\frac{\mu}{2} - \xi(r)\right) \tag{7}$$

$$\omega(r) = \pm\left(\frac{\mu}{2} + \xi(r)\right) \tag{8}$$

$$r_B = \frac{D*Cos\alpha}{2Sin\left(\frac{\mu}{2} - \alpha\right)} \tag{9}$$

2.6. Interference Coefficient

Interference coefficient indicates the power loss occurred by interference effects between upwind and downwind turbine. Weibull distribution, power curve and wind intensity interference coefficients are used to calculate interference coefficient [7].

$$I_{ij}^{d} = \Psi(r,\theta)\left(\int_0^\infty P(u_i)F_i^d(u_i)du_i - \int_0^\infty P(u_j)F_j^d(u_j)du_j\right) \tag{10}$$

I_{ij}^{d} indicates the power loss magnitude between upwind 'i' and downwind turbine 'j' for specific wind direction d. $F_i^d(u_i)$, $F_j^d(u_j)$ are weibull distribution and $P(u_i)$, $P(u_j)$ are power values for both downwind and upwind wind turbines for wind direction 'd'. According to previous calculations, the interference coefficient is used to optimize the output power of a wind farm with respect to proximity and interference constraints. To do that, as it was mentioned before, a mixed integer linear programming model is used to find the optimal numbers of wind turbines and also the best locations of them to maximize the captured power. All possible wind directions are taking into account to find WIIF for each wind sector between wind turbines. Besides that, I_{ij}^{d} is calculated for each downwind, upwind turbine pairs to find the total power loss of the wind farm.

2.7. Optimization Model

The following presented optimization model shows a maximization formula for wind farm's output power which is based on power losses in a wind farm due to interferences between wind turbines. This formula helps to find an overall captured power by using a MILP in order to find the most optimum solution [10].

Maximise

$$\sum_{i\in V, d\in S} w_i^d (P_i^d x_i - Z_i^d) \tag{11}$$

Subject To

$$\sum_{i\in q} x_i \le 1 \qquad q \in Q$$

$$I_{ij}^d (x_i + x_j - 1) \le Z_i^d \qquad i \in V, d \in S, j \in L_i^d$$

$$\sum_{i\in V} x_i \le b$$

$$x_i \in \{0,1\}, Z_i^d \ge 0 \qquad i \in V, d \in S$$

w_i^d denotes the time fraction for wind blowing in d direction. Z_i^d shows the magnitude of power loss for 'd' direction in 'i' downwind turbine because of the interference generated by upwind turbine. Moreover, x_i is used to show whether there is a turbine at 'i' location or not. If a turbine exists in 'i' location then x_i is one, otherwise, it is zero. Here, V indicates the set of possible turbine locations and L_i^d denotes the set of possible turbine locations that would cause interference at 'i' if turbines were placed in both locations for each possible turbine location i∈V [10]. Q is related to proximity constraint and indicates the collection of maximal sets of turbine locations for which at most one location can be chosen due to the minimum separation distance imposed by the turbine manufacturer [10]. And 'b' denotes the maximum number of turbines that can be placed in the wind farm. Below is the summary of the optimization process;

- Superimpose a grid onto the wind farm. The intersections of the grid define the set of possible turbine locations (n nodes).
- For every possible turbine location (grid point) calculate the interference intensity factor that a turbine in that location would cause on all other locations.
- MIP will choose best locations for the number of turbines that is specified.
- The rotor blade faces the primary wind direction at all times, so we don't need to consider the movement of the turbine. It can be assumed that whenever the wind is blowing, the turbine will be facing it.
- Calculate power values for each node.
- Wind interference coefficients for each edge connecting each node will be found.

Repeat the calculations (power, interference etc) 12 times (since there are 12 wind directions).

3. Results

For the current work, Vestas V80 wind turbine was used during the optimization process. Results were found according to a wind rose including twelve different directions. The Table 1 contains the desired data to do the calculations.

Table 1. Sector-Wise Weibull A- and k-Parameter

Sector	Direction(°)	f(%)	A(m/s)	k
N	345-15	8,1	7,85	1,80
NNE	15-45	21,6	9,91	2,73
ENE	45-75	15,3	7,33	2,46
E	75-105	9,4	6,23	2,40
ESE	105-135	2,9	4,47	1,61
SSE	135-165	1,5	3,14	1,46
S	165-195	2,1	3,78	1,40
SSW	195-225	5,7	8,28	1,54
WSW	225-255	16,9	11,74	2,24
W	255-285	9,0	8,59	1,76
WNW	285-315	4,3	4,72	1,62
NNW	315-345	3,3	5,32	1,50
All Data		100	8,34	1,81

A wind farm with a 2.16 km* 2.16 km dimension was used in the optimization process. According to these dimensions six different grid spacings were analyzed to find the best distance between grid points. Before that, based on proximity constraint maximum numbers of turbines that can be placed in a wind farm were found for each model. Table 2 indicates the results based on various distance values between grid points in a wind farm. As the distance between wind turbines increases, the number of turbines that can be placed in this hypothetical wind farm is decreasing. All the optimization processes were done on Matlab and LINGO 13.0 on a 2.26 GHz Packard bell with 4.00 GB RAM. The next table shows the maximum power and run time for GA and Lingo.

Table 2. Optimization Solutions of Different Grid Spacings

Grid Numbers	Distance btw Grids (meter)	Number of located turbines	Best objective value by GA (MWh)	Best objective value by Lingo (MWh)	Run Time for GA (seconds)	Run Time for Lingo (seconds)
5*5	540	9	15.17	15.16999	31.4355	1
6*6	432	9	11.1526	11.15260	69.4110	1
7*7	360	16	21.4991	21.49905	79.6094	14
8*8	308.57	16	17.1499	17.14994	86.7006	94
9*9	270	25	25.0375	25.03748	204.0375	315
10*10	240	25	21.8320	21.83201	148.4525	11713

*3.1. 9*9 and 10*10 Grid Form Results*

In

Figure ***4***, placements of wind turbines are shown for two different grid spacing which are 9*9 and 10*10. According to Table 2, maximum power for 9*9 and 10*910 forms are 25.0375MWh and 21.8320MWh respectively which are the highest objective values.

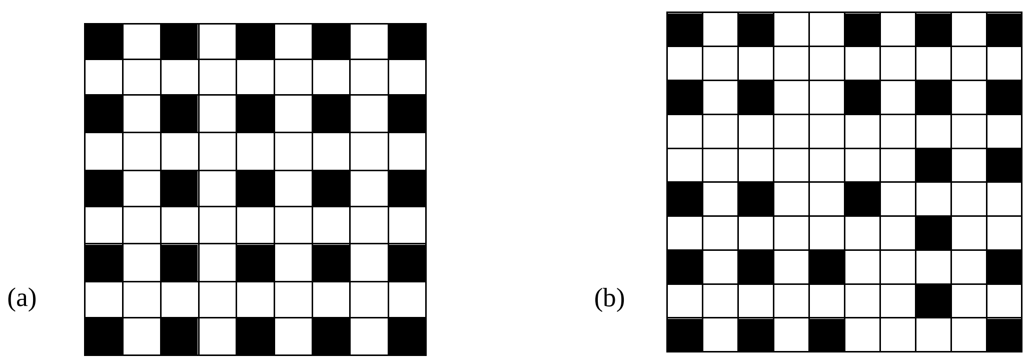

*Figure 4. Wind Turbines Placement in the Wind Farm for (a) 9*9 and (b)10*10 Grid Forms*

3.2. Disscusion

The above graphs consist of black and white squares which depict wind turbine placements. The black squares represent the places where a turbine is located and white squares show that there is not any turbine in that points. It is expected that the produced power for each located wind turbine will not be the same for different points since the wind speed frequency is not the same for all directions.

The maximum located turbines are 25 for both cases. However, the distance between grid points or the possible turbine locations is higher in 9*9 grid spacing than 10*10 form. Therefore, maximum power production is higher in 9*9 form than the 10*10 case since the wake effect is reduced. So, to obtain the maximum power output the distance between grids must be 270m or 3.375 turbine diameter. In the next page, Table 3 shows the output power for each wind turbine in the 9*9 grid spacing. It can be seen that, the maximum power production occurs in point (9,9) which is due to the high speed frequency in 30 (NNE) and 60 (ENE) degree directions. Moreover, it is obvious that for most of the points inside of the wind farm, power values are lower than power values in out border of the wind farm. This is due to the high power loss or high interference effect inside of the wind farm.

*Table 3. Power Output of Each Wind Turbine for 9*9 Case*

(x,y)	Power(MWh)
(1,9)	1.2930
(3,9)	1.1174
(5,9)	1.1134
(7,9)	1.1973
(9,9)	1.3406
(1,7)	1.1645
(3,7)	0.8538
(5,7)	0.7753
(7,7)	0.9544
(9,7)	1.1898
(1,5)	1.1695
(3,5)	0.7862
(5,5)	0.7390
(7,5)	0.8687
(9,5)	1.1258
(1,3)	1.2104
(3,3)	0.9264
(5,3)	0.6673
(7,3)	0.7233
(9,3)	0.9784
(1,1)	1.2867
(3,1)	1.0361
(5,1)	0.7794
(7,1)	0.7956
(9,1)	0.9448
Total	**25.0375(MWh)**

Conclusions and Future Work

As a conclusion, numerical solution for micro-siting of the wind farm was found for various models. Lingo and GA gives the same answers; however, solving times for Lingo is much larger than GA. On the other hand, while LINGO guarantees the most optimum solution to find the best objective value in GA, one has to run it several time. Therefore, if time is matter GA seems a better program than LINGO. Moreover, distance between wind turbines was changed and different grid forms were superimposed into the wind farm with 2.16 km*2.16 km dimension. Based on the results, as the distance between grids get larger the best object value also increases. However, as the wind farm has a fixed value, there is a limitation on numbers of possible turbine locations. It means that 5*5 grid form has the biggest spacing distance among the other forms; however, due to the proximity constraint only 9 turbines can be located in the wind farm. So, the maximum power output is not as high as in 9*9 grid form. Therefore, 270 m distance between grids seems to be the best form for the optimization of this wind farm. For future studies, a three dimensional terrain can be analyzed by using the presented optimization model. Moreover, different turbine types may be used in a single wind farm to compare the results and find the best wind turbines for a single wind farm.

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