A Hybrid Direction of Arrival Estimation on Uniform Linear Antenna Arrays

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Abstract:

In this study, a new hybrid algorithm for uniform linear antenna (ULA) array was composed by using First Order Forward Prediction (FOFP) and the Minimum Variance Distortionless Response (MVDR) algorithms together. FOFP and MVDR algorithms were discussed as incident signal direction prediction algorithms. The powerful and weak points of these two algorithms were explained. Some scripts were written by using MATLAB. 5 antennas which were spaced with half wavelength forms an ULA array. Its output data was used as input data of the three algorithms mentioned above. The results obtained from the hybrid algorithm and the individual results of MVDR and FOFP algorithms were compared under the same conditions. An improvement in the proposed hybrid algorithm is observed.

Key words: Direction of Arrival Estimation, DOA, Smart Antennas, Antenna Arrays.

1. Introduction

Antenna arrays have been used in many fields such as radar, sonar, communications, seismic data processing, and so on. Antenna arrays with adaptive signal processing known also as smart antenna have found wide application area in third-generation (3G) mobile systems for detecting the mobile users' position by using DOA estimation techniques. Adaptive antenna arrays improve the performance of cellular communication system thanks to the ability of preventing co-channel fading and generating low-side lobes interferences [1-5].

Besides, DOA estimation plays an important role in sonar and radar surveillance systems due to its facility of scanning with stationary antenna. In literature there are many antenna array installation models and algorithms for DOA estimation. Each model has some advantages and disadvantages [6,7,8].

In this paper, a hybrid algorithm combining First Order Forward Prediction (FOFP) algorithm and the Minimum Variance Distortionless Response (MVDR) algorithm is introduced.

2. Materials and Method

The number of antenna elements in ULA is assumed as *N*. Because of characteristics of ULA, the distance between antenna elements is equal to *d*. The structure of ULA is shown in Figure 1 [9].

The number of plane wave incident to the antenna array (that is number of sources in the medium) is assumed as *M*. The receiving antenna outputs including the noise is

$$\mathrm{x}_n(\mathrm{k}) = \mathrm{A}(\vartheta_m)\, s_m(\mathrm{k}) + n_n(\mathrm{k}) \qquad (1)$$

where the symbols were descripted the following Table 1.

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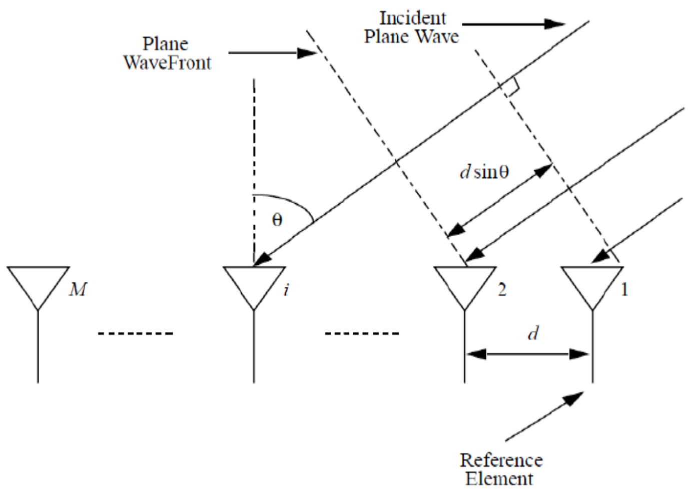

Figure 1. ULA structure

Table 1. Descriptions of the symbols in the Equation (1)

Symbol	Description	Size
k	Discrete-time variable 1 to K	K × 1
$A(\vartheta_m)$	Rotating matrix	N × M
S_m(k)	Signal matrix	M × K
n_n	Noise matrix	N × K
x_n(k)	Antenna output matrix	N × K
K	Registered sample count	

Here the angles θ_1 , θ_2 , ... , θ_M are the information to be obtained [10-12].

In this study, MVDR algorithm and FOFP algorithm are used to obtain the hybrid algorithm for DOA estimation. The hybridizing operation is applicable to any two DOA algorithms whose estimation methods are different from the other. The MVDR and FOFP algorithms are chosen because of their lower calculation costs than the others. While FOFP algorithm is one of the linear prediction techniques, MVDR algorithm is statistical learning method for parameter estimation.

2.1. MVDR Algorithm

MVDR is a ML algorithm which rely on that all of the incident signals except for the concerned direction are spurious signal. Its aim is to maximize signal interference ratio (SIR) without any distortion in the phase and amplitude of the concerned signal [13 - 17]. The principle of the method of ML is to determine mentioned magnitudes of the distribution parameters maximizing the likelihood function, or rather, the log-likelihood function for analytical convenience [18].
The results of this algorithm are presented as pseudo spectrum. It has probability values about the individual angles. Pseudo spectrum equation of this method is as follows:

$$\mathrm{P}(\vartheta) = \frac{1}{\mathrm{a}^{\mathrm{H}}(\vartheta) R_x^{-1}\ \mathrm{a}(\vartheta)} \qquad (2)$$

Table 2. Descriptions of the symbols in the Equation 2

Symbol	Description	Size
$\mathrm{P}(\vartheta)$	Pseudo-spectrum of incident signal -180 to 180	3600×1
$\mathrm{a}(\vartheta)$	Rotating matrix	N × M
R_x	Covariance matrix	N × N
$(\ )^{\mathrm{H}}$	Conjugate (Hermitian) transpose	M × N
$(\ )^{-1}$	Invers of a matrix	N × N

In this algorithm, if sources do not have different properties (in frequency, phase and amplitude) from each other, results significantly worsen [19]. The effect of decreasing SNR of incident signal causes the expanding of peaks in prediction spectrum and the shift of them from their actual position [20]. For this reason, to distinguish the signals located close to each other exactly, SNR rate need to be above the prediction threshold [21].

2.2. FOFP Algorithm

Newton predictors (NP) are very attractive because of having low calculation complexity in revealing polynomial and the simple design [22]. There are two smoothed version of the original NP. These are Linear Smoothed Newton (LSN) and the Median Smoothed Newton (MSN) [24]. It was introduced a recursively expansion to increase the applicability of LSN and so Recursive Linear Smoothed Newton Predictor (RLSN) emerged [22]. The simplest form of RLSN predictor is First Order Forward Predictor (FOFP) which is one step forward ramp predictor. Pseudo spectrum equation of FOFP used for DOA estimation is as follows:

$$\mathrm{P}(\vartheta) = \frac{u_1^H R_x^{-1} u_1}{\left|u_1^H R_x^{-1} \mathrm{a}\right|^2} \qquad (6)$$

Table 3. Descriptions of the symbols in the Equation (6)

Symbol	Description	Size
$\mathrm{P}(\vartheta)$	Pseudo-spectrum of incident signal -180 to 180	3600×1
$\mathrm{a}(\vartheta)$	Rotating matrix	N × M
R_x	Covariance matrix	N × N
u_1	A column matrix of inputs	1 × M

In this algorithm noise is not included in calculation. For low SNR values, the noise increases the errors because there is not any recursion in FOFP. For this reason, high-order and recursive (RLSN) predictions should be used for signals with low SNR, but this increases the processing cost [22].

3. Hybrid Structure

In this study, the aim was to achieve better results for low SNR values by using data of the two algorithms. There were 5 antenna elements in antenna array. All information obtained from antenna was stored in shift registers. The length of shift registers was 100. In this situation, dimension of the matrix was 5×100. This was the data in the X register which was one of the inputs for calculation of MVDR and FOFP algorithms, at the 1st and 2nd line of MATLAB codes. The other input for estimation algorithms d was the distance between the two antennas in the array, as wavelength.

The dimensions of the pseudo spectrum results for both FOFP and MVDR were 3600×1. The results were entered in the hybridizing stage. In hybridizing stage dimensions of inputs must be the same length. Similar to other pseudo spectrums, the dimension of output was also 3600×1. The functional block diagram of the hybrid algorithm was shown in Figure 2.

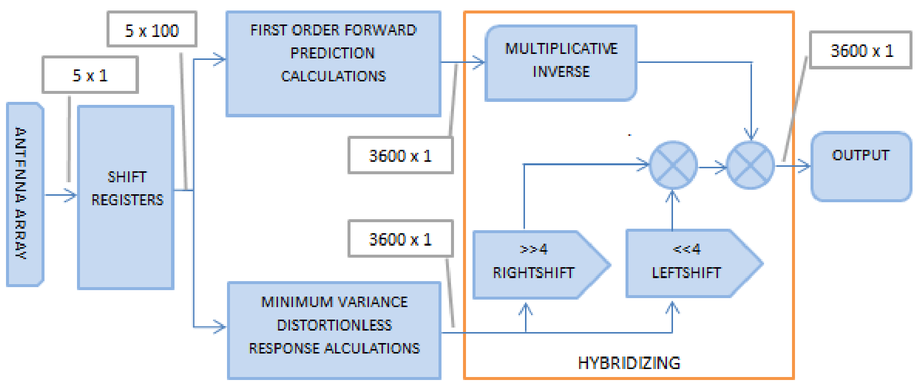

Figure 2. Hybrid Algorithm Block Diagram

For hybridizing, MVDR algorithm results were used as denominator. As a start of hybridizing in 4^{th} 5^{th} 7^{th} and 8^{th} lines, the MVDR and FOFP results which were obtained at first and second steps were normalized. This makes the minimum value of the series to 0 and the maximum value to 1. A stationary value assumed as 10 percent of maximum of MVDR series was added to avoid asymptotes. Two variables with zero values and in the same size with MVDR were defined at 10^{th} and 11^{th} steps.

The situation of MVDR which was shifted 4 units to the right and 4 units to the left according to its position in spectrum at 13^{th} and 14^{th} steps were assigned to the variables defined at 10^{th} and 11^{th} step. Each point in pseudo spectrum in the proposed algorithm was 0.1 degree. It was assumed that there was not any signal at 4 unit left and right of the incident signal.

Then, to obtain the denominator (K_MVDR) these two variables were multiplied one by one with ".*" at 16^{th} step. By multiplying the 4 units right and left shifted series, the series called K_MVDR was obtained. At step 17 to avoid asymptotes at the end of the dividing process, 10%

of the maximum value of K_MVDR was added to K_MVDR variable. To achieve the result of hybrid structure at 19th step, the FOFP results were divided to K_MVDR.

MATLAB codes of these steps are below.

```
1.    [~, FOFP]=FOFP_DOAE(X,d);
2.    [~, MVDR]=MVDR_DOAE(X,d);
3.
4.    FOFP=FOFP-min(FOFP);
5.    FOFP=FOFP./max(FOFP);
6.
7.    MVDR=MVDR-min(MVDR);
8.    MVDR=MVDR./max(MVDR);
9.
10.   K_MVDR1=double(zeros(size(MVDR)));
11.   K_MVDR2=double(zeros(size(MVDR)));
12.
13.   K_MVDR1(4:3601)=data3(1:3598);
14.   K_MVDR2(1:3598)=data3(4:3601);
15.
16.   K_MVDR=K_MVDR1.*K_MVDR2;
17.   K_MVDR=K_MVDR+max(K_MVDR)/10;
18.
19.   HYBRD=FOFP./K_MVDR;
20.
```

4. Discussion

To test this new hybrid algorithm, an ULA which had 5 isotropic antennas placed apart of each other by half wavelength was designed. All of signal sources were unity amplitude and frequency of 1 MHz. Phase Shift Keying (PSK) was used to modulate randomly produced data signals on each source’s carrier. 100 samples were taken from the antenna outputs by 10 MHz sampling frequency. The simulation set was run to test hybrid algorithm in MATLAB.

This test was done to determine distinguishability for close arrival signals. It was accepted that the angles of signal sources to the antenna array were -41, -24, 20 and 30 degrees and the signal source at -24 degrees moves towards -40 degrees. These simulations were run in MATLAB for the case of four different incident signals with frequency modulation at 40 dB SNR value. The used antenna array was the half wavelength spaced ULA which was consist of 5 isotropic elements.

For this situation, the result of MVDR algorithm was shown in Figure 3, the result of FOFP algorithm was shown in Figure 4 and the result of Hybrid algorithm was shown in Figure 5.

In the Figure 3, while the signal at the -24 degrees was moving to -40 degrees, footprint of MVDR result was drawn. Likewise, footprints of FOFP and Hybrid algorithms were drawn in the Figure 4 and Figure 5, respectively. It was able to seen in Figure 5, the footprints of incident signals were exactly distinguished but in Figure 3 and Figure 4 they were not capable to distinguish the incident signals when closer than certain angle.

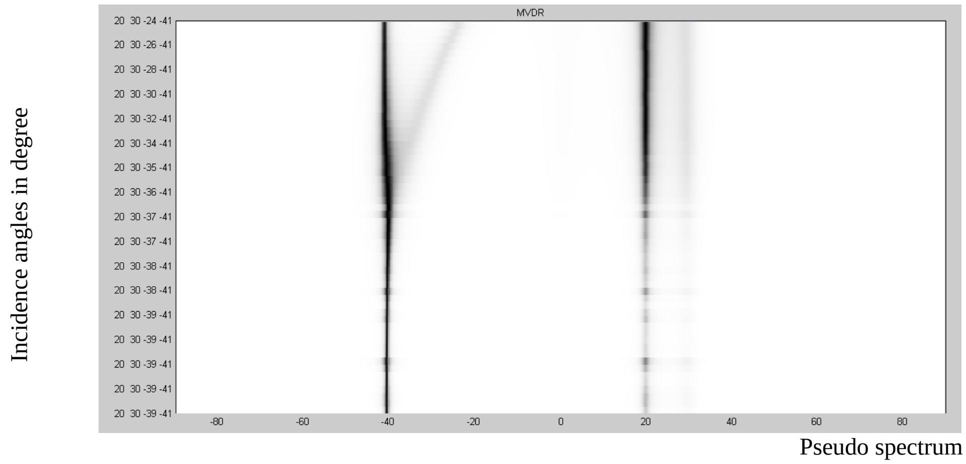

Figure 3. MVDR Algorithm Result

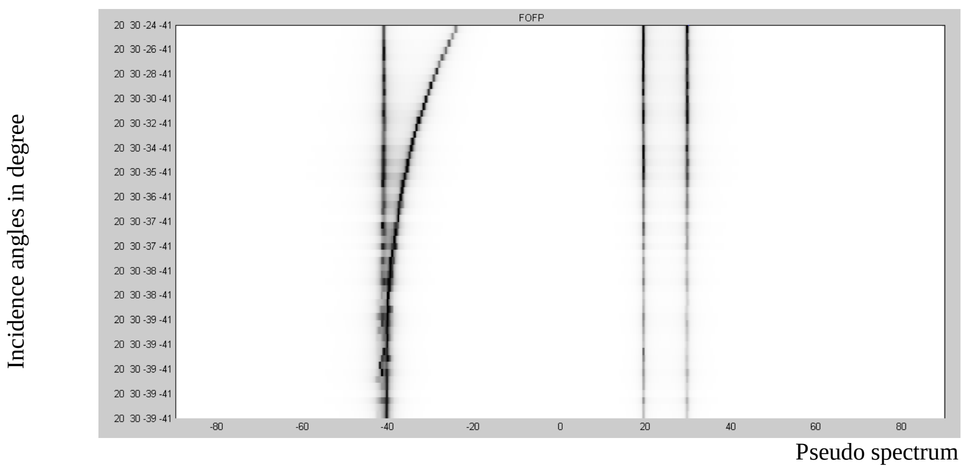

Figure 4. FOFP Algorithm Result

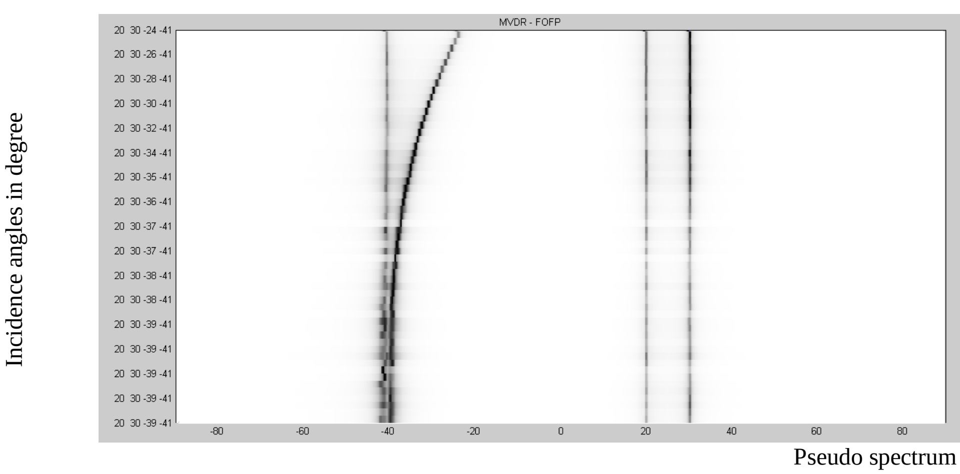

Figure 5. MVDR-FOFP Hybrid Structure Result

The same results in the Figure 3, 4 and 5 were presented as a different type in the Figure 6. It was zoomed in changing area of the pseudo spectrum. At the left side of each row in Figure 6, there was information about the angles of incident signals. At each row the results of MVDR, FOFP and HYBRID algorithms for given angles were presented separately.

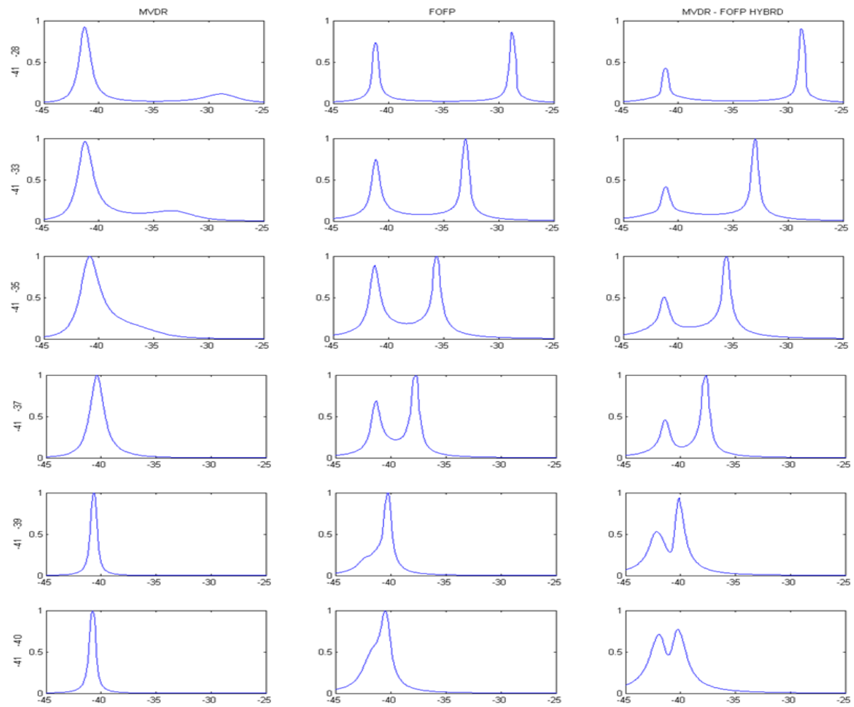

Figure 6. Results of the same incoming signals of MVDR, FOFP and Hybrid Structure

As seen in the Figure 6 the last row where MVDR algorithm was capable to distinguish the incident signals was second one. The angles of incident signals at this row were -41 and -33. That means MVDR algorithm was not able to distinguish the signals closer than 8 degrees. FOFP algorithm had better results than MVDR. The last row where FOFP algorithm is capable to distinguish the incident signals was fourth one. The angles of incident signals at this row were -41 and -37. That means FOFP algorithm was not able to distinguish the signals closer than 4 degrees. Hybrid algorithm was able to distinguish the signals closer than 1 degree but it had a small shift at detecting angles of incident signals. The improvement of the hybrid algorithm was the ability of distinguishing closer signals.

The angles of arrivals obtained from the simulation results were presented in Table 4. The Root Mean Square Error (RMSE) is regarded as a measurement of error. RMSE could be expressed as [22]:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (X_{obs,i} - X_{model,i})^2}{n}} \quad (7)$$

Table 4 The results of the simulation

Angles Of Arrivals	The Values Obtained From MVDR	The Values Obtained From FOFP	The Values Obtained From HYBRD
-41 / -28 20 / 30	-41,33 / -28,83 20,27 / 29,62	-41,16 / -26,16 19,43 / 29,54	-41,25 / -28,75 19,88 / 30,11
-41 / -33 20 / 30	-41,25 / -33,50 20,27 / 29,62	-41,16 / -33,00 19,43 / 29,54	-41,25 / -33,00 19,88 / 30,11
-41 / -35 20 / 30	-40,92 / -36,42 20,27 / 29,62	-41,25 / -35,75 19,43 / 29,54	-41,25 / -35,50 19,88 / 30,11
-41 / -37 20 / 30	-40,41 / -40,41 20,27 / 29,62	-41,33 / -37,91 19,43 / 29,54	-41,33 / -37,58 19,88 / 30,11
-41 / -39 20 / 30	-40,83 / -40,83 20,27 / 29,62	-40,41 / -40,41 19,43 / 29,54	-42,08 / -40,08 19,88 / 30,11
-41 / -40 20 / 30	-40,75 / -40,75 20,27 / 29,62	-40,41 / -40,41 19,43 / 29,54	-41,83 / -40,56 19,88 / 30,11

In the Table 5, the RMS errors of the difference between two angles of arrivals relative to the real values were listed for each situation.

Table 5 The RMS errors

The Angle Differance of The Closest Arrivals	RMSE for MVDR	RMSE for FOFP	RMSE for HYBRID
10	0,046	0,094	0,027
8	0,042	0,035	0,017
6	0,150	0,049	0,028
4	0,580	0,084	0,039
2	0,579	0,577	0,016
1	0,579	0,577	0,157

The graphically presentation of these RMS errors were shown at the Figure 7.

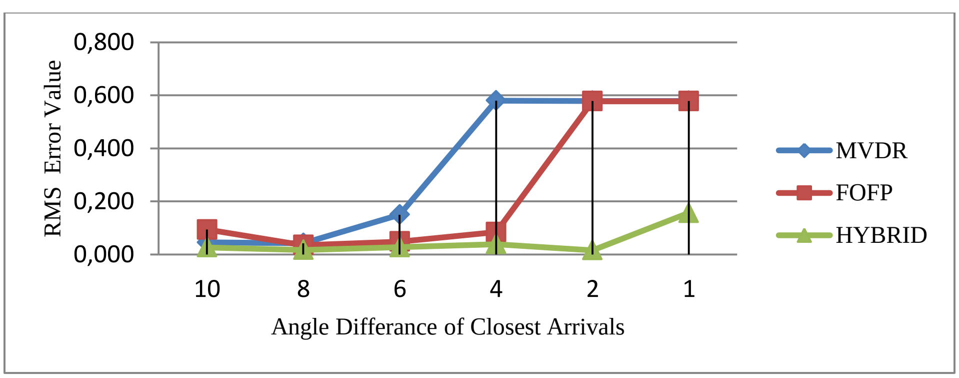

Figure 7. The RMS Errors versus Angle Difference of Closest Arrivals

Conclusions

The majority of spectral prediction techniques were used to carry out the sources or possible reflections in the environment. For this reason, accuracy in signal direction prediction was very important. In this study, a new hybrid algorithm was developed by using MVDR and FOFP algorithms. As mentioned before, noise immunity of these algorithms was low.

As seen in the simulation results, the hybrid algorithm was more successful than individual FOFP and MVDR algorithms in distinguishing two signals that were close to each other at lower SNR values.

On the other hand, in MATLAB simulations, time requirements of MVDR, FOFP and HYBRID algorithms were 40.8, 57.5 and 84.9 milliseconds, respectively. It was able to seen that the cost of hybrid algorithm was higher than individual algorithms. This situation may pose a problem for applications requiring high refresh rate.

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