Blow up of solutions to an inverse problem for a quasilinear parabolic equation

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Abstract

In this study we consider an inverse problem for quasilinear parabolic equation with type power nonlinearity. Sufficient conditions on initial data and for blow up result is obtained with positive initial energy. Over determination condition is given integral form. To get the blow up result for this nonlinear inverse parabolic equation we use the concavity of positive function. The life span of solution is also computed.

Keywords*:* Blow-up, inverse problem, quasilinear parabolic equation.

1 Introduction

Inverseproblemsaretheproblemsthatconsist of finding an unknownproperty of an object, or a medium, fromtheobservation of a response of thisobject, ormedium, to a probingsignal. Thus, thetheory of inverseproblemsyields a theoreticalbasisforremotesensingandnon-destructiveevaluation. Forexample, if an acousticplanewave is scatteredby an obstacle, andoneobservesthescatteredfield far fromtheobstacle, or in someexteriorregion, thentheinverse problem is tofindtheshapeandmaterialproperties of theobstacle. Suchproblemsareimportant in identification of flyingobjects (airplanesmissiles, etc.), objectsimmersed in water (submarines, paces of fish, etc.), and in manyothersituations.

Ingeophysicsonesends an acousticwavefromthesurface of theearthandcollectsthescatteredfield on thesurfaceforvariouspositions of thesource of thefieldfor a fixedfrequency, orforseveralfrequencies. Theinverse problem is tofindthesubsurfaceinhomogeneities. Intechnologyonemeasurestheeigenfrequencies of a piece of a material, andtheinverse problem is tofind a defect in thismaterial, forexample, a hole in a metal. Ingeophysicstheinhomogeneity can be an oildeposit, a cave, a mine. Inmedicine it may be a tumor, orsomeabnormality in a human body.

We now consider the following inverse problem for a quasilinear parabolic equation

$$u_t - \nabla.[(k_1 + k_2|\nabla u|^{m-2})\nabla u)] + h(u, \nabla u) - |u|^{p-2}u = F(t)w(x) \quad (1)$$
$$u(x,t) = 0,\ x \in \partial\Omega, t > 0 \quad (2)$$
$$u(x,0) = u_0,\ x \in \Omega \quad (3)$$
$$\int_\Omega u(x,t)w(x)dx = 1,\ t > 0 \quad (4)$$

where$\Omega \subseteq \mathbb{R}^n, n \geq 1$ is bounded domain with a sufficiently smooth boundary $\partial\Omega. \rho, k_1$ and k_2 positive constants and $p > m \geq 2$. Also assume that $w(x)$ is given function satisfying

$$\int_\Omega w^2(x)dx = 1, \quad w \in H^m(\Omega) \cap H_0^1(\Omega) \cap L^p(\Omega)\ , m \geq 2 \qquad (5)$$

The inverse problem consists of finding a pair of functions $\{u(x,t), F(t)\}$ satisfying (1)-(4) when

$$\int_\Omega u_0 w dx = 1, \quad u_0 \in H_0^1(\Omega) \cap L^p(\Omega) \qquad (6)$$

and$h(u, \nabla u)$ is continuous function which have the relation

$$|h(u,\nabla u)| \leq K\left(|u|^{\frac{p}{2}} + |\nabla u|^{\frac{m}{2}}\right), K > 0. \qquad (7)$$

Additional information about the solution to the inverse problem is given in the form of the integral over determination condition (4). Temperature $u(x,t)$ is averaging by function w over domain Ω[1].

Existence and uniquesness of solutions to an inverse problem for parabolic equations are studied several authors [3,5,6,7,]. Asymptotic stability of solutions to inverse problem for parabolic equations are investigated in some studies [1,5,8,9].

Global nonexistence and blow up results for nonlinear parabolic equations is discussed in some papers [10,11]. But less is known about inverse problem for nonlinear parabolic equations. Eden and Kalantarov [7] studied the following problem;

$$u_t - \Delta u - |u|^p u + b(x,t,u,\nabla u) = F(t)w(x), \qquad p > 0.$$

In this work, we consider blow up results in finite time for solutions to inverse problem for nonlinear parabolic equation (1)-(4) with weight function $w(x)$. The proof of our technique is similar to the one in [11].

In this paper, we use the following notations;
$\|u\| = \|u\|_{L_2(\Omega)}$, $\|u\|_p = \|u\|_{L_p(\Omega)}$ are usual the lebesque spaces, $(u,v) = \int_\Omega uv dx$ is the inner product,

$$ab \leq \varepsilon a^2 + \frac{1}{4\varepsilon} b^2$$

is the weighted arithmetic-geometric inequality for $a, b > 0$ and

$$ab \leq \beta a^q + C(p,\beta) b^{q'}$$

is the Young`s inequality with $\frac{1}{q}+\frac{1}{q'}=1, C(q,\beta)=\frac{1}{q'(q\beta)^{q'/q}}$.

Let us note the following lemma known as "Ladyzhenskaya-Kalantarov lemma". It is good tool to obtain the blow up results for dynamical problems.

Lemma 1*Suppose that a positive, twice differentiable function $\psi(t)$ satisfies for $t>0$ the following inequality*

$$\psi\psi''-(1+\gamma)(\psi')^2\geq -2M_1\psi\psi'-M_2\psi^2$$

where $\gamma>0, M_1, M_2\geq 0$. If $(0)>0, \psi^{'}(0)>-\gamma_2\gamma^{-1}\psi(0)$, and $M_1+M_2>0$, then $\psi(t)$*tends to infinity* as $t\to t_1\leq t_2$.

$$t_2\leq\frac{1}{2\sqrt{M_1^2+\gamma M_2}}ln\frac{\gamma_1\psi(0)+\gamma\psi'(0)}{\gamma_2\psi(0)+\gamma\psi'(0)}$$

where$\gamma_1=-M_1+\sqrt{M_1^2+\gamma M_2}$, $\gamma_2=-M_1-\sqrt{M_1^2+\gamma M_2}$.

Proof (see [4])

2 Blow-up Result

Theorem 1 *Suppose that the conditions (3) and (4) are satisfied. Let $\{u(x,t),F(t)\}$ be the solution of inverse problem (1)-(4). Assume the following conditions are valid:*

$$\gamma=\sqrt{1+\beta}-1,\ \beta\epsilon(0,\alpha),\ \alpha=\frac{p+m-4}{8}, \lambda=\frac{K^2(m+pk_2)(1+\alpha)}{k_2(p-m)(\alpha-\beta)} \quad (8)$$

$$E(0)=-\frac{\lambda}{2}\|u_0\|^2-\frac{k_1}{2}\|\nabla u_0\|^2-\frac{k_2}{m}\|\nabla u_0\|_m^m+\frac{1}{p}\|u_0\|_p^p>0 \quad (9)$$

$$4(1+2\alpha)E(0)-\frac{2\lambda(1+\gamma)^2}{\gamma}\|u_0\|^2>D_3 \quad (10)$$

where

$$D_3=\left[\frac{8K^2(m+pk_2)}{k_2(p-m)}\|w\|^2+\frac{4k_2}{m}\left(\frac{8(m-1)}{(p-m)}\right)^{m-1}\|\nabla w\|_m^m+\frac{4}{p}\left(\frac{8(p-1)}{(p-m)}\right)^{p-1}\|w\|_p^p+\frac{4k_1}{p+m-4}\|\nabla w\|^2\right] (11)$$

Then there exists a finite time t_1 such that

$$\|u\|^2\to+\infty as t\to t_1^-.$$

Proof: For $\lambda > 0$, we make the transformation $u(x,t) = e^{\lambda t}v(x,t)$in (1) and we obtain the equation

$$v_t - \nabla.\left[(k_1 + k_2 e^{\lambda(m-2)t}|\nabla v|^{m-2})\nabla v\right] + \lambda v + e^{-\lambda t}h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right) - e^{\lambda(p-2)t}|v|^{p-2}v = e^{-\lambda t}F(t)\varphi(x) \quad (12)$$

with the boundary condition

$$v(x,t) = 0,\ x \in \partial\Omega, t > 0\ , \quad (13)$$

the initial condition

$$v(x,0) = u_0,\ x \in \Omega\ , \quad (14)$$

and the integral over determination condition

$$\int_\Omega v(x,t)w(x)dx = e^{-\lambda t},\ t > 0 \quad (15)$$

Let us multiply the equation (12) by $v_t$in $L^2(\Omega)$, we get the relation

$$\frac{d}{dt}\left[\frac{\lambda}{2}\|v\|^2 + \frac{k_1}{2}\|\nabla v\|^2 + \frac{k_2}{m}e^{\lambda(m-2)t}\|\nabla v\|_m^m - \frac{1}{p}e^{\lambda(p-2)t}\|v\|_p^p\right] - \frac{k_2\lambda(m-2)}{m}e^{\lambda(m-2)t}\|\nabla v\|_m^m$$

$$+\|v_t\|^2 + \frac{\lambda(p-2)}{p}e^{\lambda(p-2)t}\|v\|_p^p + e^{-\lambda t}\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), v_t\right) = -\lambda e^{-2\lambda t}F(t) \quad (16)$$

Now, multiply the equation (12) by $w$in $L^2(\Omega)$ and use over determination condition (15), then we obtain

$$F(t) = k_1 e^{\lambda t}(\nabla v, \nabla w) + k_2 e^{\lambda(m-1)t}(|\nabla v|^{m-2}\nabla v, \nabla w) + \left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), w\right)$$

$$-e^{\lambda(p-1)t}(|v|^{p-2}v, w) \quad (17)$$

Substituting equation (17) in equation (16) we get the relation

$$-\frac{d}{dt}j(t) - \frac{\lambda k_2(m-2)}{m}e^{\lambda(m-2)t}\|\nabla v\|_m^m + \|v_t\|^2 + \frac{\lambda(p-2)}{p}e^{\lambda(p-2)t}\|v\|_p^p$$

$$= e^{-\lambda t}\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), v_t\right) - \lambda e^{-2\lambda t}\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), w\right) - \lambda k_2 e^{\lambda(m-3)t}(|\nabla v|^{m-2}\nabla v, \nabla w)$$
$$+\lambda e^{\lambda(p-3)t}(|v|^{p-2}v, w) - \lambda k_1 e^{-\lambda t}(\nabla v, \nabla w) \quad (18)$$

where$E(t) = \frac{1}{p}e^{\lambda(p-2)t}\|v\|_p^p - \frac{k_2}{m}e^{\lambda(m-2)t}\|\nabla v\|_m^m - \frac{\lambda}{2}\|v\|^2 - \frac{k_1}{2}\|\nabla v\|^2$.

Use the property of function $h(e^{\lambda t}v, e^{\lambda t}\nabla v)$ given by (7) and then apply the weighted arithmetic-geometric inequality to the first term on the right side of (18) with $a = e^{\lambda(p-2)t/2}\|v\|_p^{p/2}$, $b = K\|v_t\|$, $\varepsilon = \frac{\lambda(p-m)}{4p}$ and $a = e^{\lambda(m-2)t/2}\|\nabla v\|_m^{m/2}$, $b = K\|v_t\|$, $\varepsilon = \frac{\lambda k_2(p-m)}{4m}$ to get the estimate

$$e^{-\lambda t}\left|\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), v_t\right)\right| \leq \frac{\lambda(p-m)}{4p}e^{\lambda(p-2)t}\|v\|_p^p + \frac{\lambda k_2(p-m)}{4m}e^{\lambda(m-2)t}\|\nabla v\|_m^m + \frac{K^2(m+pk_2)}{\lambda k_2(p-m)}\|v_t\|^2 \quad (19)$$

We can obtain similar result for the second term on the right side of (18) with $a = e^{\lambda(p-2)t/2}\|v\|_p^{p/2}$, $b = \lambda K\|w\|$, $\varepsilon = \frac{\lambda(p-m)}{8p}$ and $a = e^{\lambda(m-2)t/2}\|\nabla v\|_m^{m/2}$, $b = \lambda K\|w\|$, $\varepsilon = \frac{\lambda k_2(p-m)}{8m}$

$$\lambda e^{-2\lambda t}\left|\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), w\right)\right| \leq \frac{\lambda(p-m)}{8p}e^{\lambda(p-2)t}\|v\|_p^p + \frac{\lambda k_2(p-m)}{8m}e^{\lambda(m-2)t}\|\nabla v\|_m^m$$

$$+\frac{2\lambda K^2(m+pk_2)}{k_2(p-m)}e^{-2\lambda t}\|w\|^2 \quad (20)$$

Apply Young's inequality to the third and fourth terms on the right side of (18) with $a = e^{\frac{\lambda(m-2)(m-1)}{m}t}\|\nabla v\|_m^{m-1}$, $b = \lambda k_2 e^{\frac{-2\lambda t}{m}}\|\nabla w\|_m$, $\varepsilon = \frac{\lambda k_2(p-m)}{8m}$ and $a = e^{\frac{\lambda(p-2)(p-1)t}{p}}\|v\|_p^{p-1}$, $b = \lambda e^{\frac{-2\lambda t}{p}}\|w\|_p$, $\varepsilon = \frac{\lambda(p-m)}{8p}$ to get the estimates respectively;

$$\lambda k_2 e^{\lambda(m-3)t}|(|\nabla v|^{m-2}\nabla v, \nabla w)| \leq \frac{\lambda k_2(p-m)}{8m}e^{\lambda(m-2)t}\|\nabla v\|_m^m + \frac{\lambda k_2}{m}\left(\frac{8(m-1)}{p-m}\right)^{m-1}e^{-2\lambda t}\|\nabla w\|_m^m \quad (21)$$

$$\lambda e^{\lambda(p-3)t}|(|v|^{p-2}v, w)| \leq \frac{\lambda(p-m)}{8p}e^{\lambda(p-2)t}\|v\|_p^p + \frac{\lambda}{p}\left(\frac{8(p-1)}{p-m}\right)^{p-1}e^{-2\lambda t}\|w\|_p^p \quad (22)$$

The last term on the right side of (18) can be estimated by weighted arithmetic-geometric inequality with $a = \|\nabla v\|$, $b = \lambda k_1 e^{-\lambda t}\|\nabla w\|$, $\varepsilon = \frac{\lambda k_1(p+m-4)}{4}$

$$\lambda k_1 e^{-\lambda t}|(\nabla v, \nabla w)| \leq \frac{\lambda k_1(p+m-4)}{4}\|\nabla v\|^2 + \frac{\lambda k_1}{p+m-4}e^{-2\lambda t}\|\nabla w\|^2 \quad (23)$$

Substituting these inequalities (19)-(23) in equation (18) we get the following relation

$$\frac{d}{dt}E(t) \geq \frac{\lambda}{2}(p+m-4)E(t) + \frac{\lambda^2}{2}(p+m-4)\|v\|^2 + \frac{k_1}{4}(p+m-4)\|\nabla v\|^2$$

$$+\left\{1 - \frac{K^2(m+pk_2)}{\lambda k_2(p-m)}\right\}\|v_t\|^2 - D_0 e^{-2\lambda t} \quad (24)$$

where

$$D_0 = \frac{2\lambda K^2(m+pk_2)}{k_2(p-m)}\|w\|^2 + \frac{\lambda k_2}{m}\left(\frac{8(m-1)}{p-m}\right)^{m-1}\|\nabla w\|_m^m + \frac{\lambda}{p}\left(\frac{8(p-1)}{p-m}\right)^{p-1}\|w\|_p^p + \frac{\lambda k_1}{p+m-4}\|\nabla w\|^2.$$

From (8) we rewrite the inequality (24) as follows

$$\frac{d}{dt}E(t) \geq \frac{\lambda}{2}(p+m-4)j(t) + \frac{\lambda^2}{2}(p+m-4)\|v\|^2 + \frac{\lambda k_1}{4}(p+m-4)\|\nabla v\|^2$$

$$+\left(\frac{1+\beta}{1+\alpha}\right)\|v_t\|^2 - D_0 e^{-2\lambda t} \quad (25)$$

Since $p+m-4>0$, the second and third terms on the right side of (25) can be omitted to get the inequality

$$\frac{d}{dt}E(t) \geq \frac{\lambda}{2}(p+m-4)j(t) + \left(\frac{1+\beta}{1+\alpha}\right)\|v_t\|^2 - D_0 e^{-2\lambda t} \quad (26)$$

Solving the differential inequality (26) with the estimation $1 - e^{-\frac{\lambda}{2}(p+m)t}$ by 1 we get

$$E(t) \geq (E(0) - D_1)e^{\frac{\lambda}{2}(p+m-4)t} + \left(\frac{1+\beta}{1+\alpha}\right)\int_0^t \|v_\tau\|^2 d\tau \quad (27)$$

where $D_1 = \frac{2D_0}{\lambda(p+m)}$. It is easy to see that $E(t) \geq e^{\frac{\lambda}{2}(p+m-4)t}(E(0) - D_1) \geq j(0) - D_1$ by assumption (11). Thus we obtain a lower bound for $E(t)$

$$E(t) \geq \left(\frac{1+\beta}{1+\alpha}\right)\int_0^t \|v_\tau\|^2 d\tau + E(0) - D_1. \quad (28)$$

Multiplying the equation (12) by v in $L^2(\Omega)$ and inserting (17) we get

$$\frac{1}{2}\frac{d}{dt}\|v\|^2 + \lambda\|v\|^2 + k_1\|\nabla v\|^2 + k_2 e^{\lambda(m-2)t}\|\nabla v\|_m^m - e^{\lambda(p-2)t}\|v\|_p^p = -e^{-\lambda t}\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), v\right)$$

$$+k_2 e^{\lambda(m-3)t}(|\nabla v|^{m-2}\nabla v, \nabla w) + e^{-2\lambda t}\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), w\right) - e^{(p-3)\lambda t}(|v|^{p-2}v, w)$$

$$+k_1 e^{-\lambda t}(\nabla v, \nabla w) \quad (29)$$

Remember the condition (7) and then apply the weighted arithmetic-geometric inequality to the first term on the right side of (29) with $a = e^{\frac{\lambda(p-2)}{2}t}\|v\|_p^{p/2}$, $b = K\|v\|$, $\varepsilon = \frac{p-m}{4p}$ and $a = e^{\frac{\lambda(m-2)}{2}t}\|\nabla v\|_m^{m/2}$, $b = K\|v\|$, $\varepsilon = \frac{k_2(p-m)}{4m}$ to get the estimate

$$e^{-\lambda t}\left|\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), v\right)\right| \leq \frac{p-m}{4p}e^{\lambda(p-2)t}\|v\|_p^p + \frac{k_2(p-m)}{4m}e^{\lambda(m-2)t}\|\nabla v\|_m^m + \frac{K^2(m+pk_2)}{k_2(p-m)}\|v\|^2 \quad (30)$$

We can find similar result for the thirthterm on the right side of (29) with$a = e^{\frac{\lambda(p-2)}{2}t}\|v\|_p^{p/2}$, $b = K\|w\|$, $\varepsilon = \frac{p-m}{8p}$ and $a = e^{\frac{\lambda(m-2)}{2}t}\|\nabla v\|_m^{m/2}$, $b = K\|w\|$, $\varepsilon = \frac{k_2(p-m)}{8m}$

$$e^{-2\lambda t}\left|\left(h\left(e^{\lambda t}v, e^{\lambda t}\nabla v\right), w\right)\right| \le \frac{p-m}{8p}e^{\lambda(p-2)t}\|v\|_p^p + \frac{k_2(p-m)}{8m}e^{\lambda(m-2)t}\|\nabla v\|_m^m$$

$$+\frac{2K^2(m+pk_2)}{k_2(p-m)}e^{-2\lambda t}\|w\|^2 \qquad (31)$$

Apply Young's inequality to the second and fourth terms on the right side of equation (29) with $a = e^{\frac{\lambda(m-2)(m-1)}{m}t}\|\nabla v\|_m^{m-1}$, $b = k_2 e^{\frac{-2\lambda}{m}t}\|\nabla w\|_m$, $\varepsilon = \frac{k_2(p-m)}{8m}$ and $a = e^{\frac{\lambda(p-2)(p-1)}{p}t}\|v\|_p^{p-1}$, $b = e^{\frac{-2\lambda}{p}t}\|w\|_p$, $\varepsilon = \frac{(p-m)}{8p}$ to get the estimates respectively;

$$k_2 e^{\lambda(m-3)t}|(|\nabla v|^{m-2}\nabla v, \nabla w)| \le \frac{k_2(p-m)}{8m}e^{\lambda(m-2)t}\|\nabla v\|_m^m + \frac{k_2}{m}\left(\frac{8(m-1)}{(p-m)}\right)^{m-1}e^{-2\lambda t}\|\nabla w\|_m^m \qquad (32)$$

$$e^{\lambda(p-3)t}|(|v|^{p-2}v, w)| \le \frac{p-m}{8p}e^{\lambda(p-2)t}\|v\|_p^p + \frac{1}{p}\left(\frac{8(p-1)}{(p-m)}\right)^{p-1}e^{-2\lambda t}\|w\|_p^p \qquad (33)$$

The last term on the right side of equation (29) can be estimated by using weighted arithmetic-geometric inequality with$a = \|\nabla v\|$, $b = k_1 e^{-\lambda t}\|\nabla w\|$, $\varepsilon = \frac{k_1(p+m-4)}{4}$.

$$k_1 e^{-\lambda t}|(\nabla v, \nabla w)| \le \frac{k_1(p+m-4)}{4}\|\nabla v\|^2 + \frac{k_1}{p+m-4}e^{-2\lambda t}\|\nabla w\|^2 \qquad (34)$$

Substitute estimates (30)-(34) to obtain the following differential inequality

$$\frac{1}{2}\frac{d}{dt}\|v\|^2 \ge \frac{p+m}{2p}e^{\lambda(p-2)t}\|v\|_p^p - \frac{k_2(p+m)}{2m}e^{\lambda(m-2)t}\|\nabla v\|_m^m - \left\{\lambda + \frac{K^2(m+pk_2)}{k_2(p-m)}\right\}\|v\|^2$$

$$-\frac{k_1(p+m)}{4}\|\nabla v\|^2 - D_2 e^{-2\lambda t} \qquad (35)$$

where

$$D_2 = \frac{2K^2(m+pk_2)}{k_2(p-m)}\|w\|^2 + \frac{k_2}{m}\left(\frac{8(m-1)}{p-m}\right)^{m-1}\|\nabla w\|_m^m + \frac{1}{p}\left(\frac{8(p-1)}{p-m}\right)^{p-1}\|w\|_p^p + \frac{k_1}{p+m-4}\|\nabla w\|^2.$$

Rewrite the inequality (35) as follow

$$\frac{1}{2}\frac{d}{dt}\|v\|^2 \ge \frac{p+m}{2}E(t) + \left[\frac{\lambda(p+m-4)}{4} - \frac{K^2(m+pk_2)}{k_2(p-m)}\right]\|v\|^2 - D_2 e^{-2\lambda t} \qquad (36)$$

Since $-D_2 e^{-2\lambda t} \ge -D_2$ and $p + m - 4 > 0$, we can write (36) as

$$\frac{1}{2}\frac{d}{dt}\|v\|^2 \geq \frac{p+m}{2}E(t) - \frac{K^2(m+pk_2)}{k_2(p-m)}\|v\|^2 - D_2 \quad (37)$$

Substituting the estimate (28) and $p + m = 4(1 + 2\alpha)$ in (37) we obtain

$$\frac{1}{2}\frac{d}{dt}\|v\|^2 \geq 2(1+2\alpha)\left(\frac{1+\beta}{1+\alpha}\right)\int_0^t \|v_\tau\|^2 d\tau + 2(1+2\alpha)(E(0) - D_1) - D_2 - \frac{K^2(m+pk_2)}{k_2(p-m)}\|v\|^2 \quad (38)$$

Since $\lambda > \frac{K^2(m+pk_2)}{k_2(p-m)}$ by assumption (8) then it follows from (38)

$$\frac{d}{dt}\|v\|^2 \geq 4(1+2\alpha)\left(\frac{1+\beta}{1+\alpha}\right)\int_0^t \|v_\tau\|^2 d\tau - 2\lambda\|v\|^2 + 4(1+2\alpha)E(0) - D_3 \quad (39)$$

where $D_3 = 4(1+2\alpha)D_1 + 2D_2$.

Now let us introduce the positive function

$$\psi(t) = \int_0^t \|v\|^2 d\tau + C_0 \quad (40)$$

whereC_0 is a positive constant will be chosen later. First and second derivatives of (40) as follows

$$\psi'(t) = \|v\|^2 = 2\int_0^t (v, v_\tau) d\tau + \|u_0\|^2. \quad (41)$$

$$\psi''(t) = \frac{d}{dt}\|v\|^2 \quad (42)$$

Apply the Cauchy-Schwarz inequality and the weighted arithmetic-geometric inequality to get an upper bound for $\psi'(t)$;

$$[\psi'(t)]^2 = 4\left[\int_0^t (v, v_\tau) d\tau + \frac{1}{2}\|u_0\|^2\right]^2 \leq 4\left[\sqrt{\int_0^t \|v\|^2 d\tau}\sqrt{\int_0^t \|v_\tau\|^2 d\tau} + \frac{1}{2}\|u_0\|^2\right]$$

$$\leq 4\left[(1+4\varepsilon)\left(\int_0^t \|v\|^2 d\tau\right)\left(\int_0^t \|v_\tau\|^2 d\tau\right) + \frac{1}{4}\left(1 + \frac{1}{4\varepsilon}\right)\|u_0\|^4\right] \quad (43)$$

Remembering the relations (40)-(43) we can estimate the term $\psi\psi'' - (1+\gamma)(\psi')^2$;

$$\psi\psi'' - (1+\gamma)(\psi')^2 \geq 4(1+\beta)\left(\int_0^t \|v\|^2 d\tau\right)\psi + [(p+m)j(0) - D_3]\psi - 2\lambda\|v\|^2\psi$$

$$-4(1+\gamma)\left[(1+4\varepsilon)\left(\int_0^t \|v\|^2 d\tau\right)\left(\int_0^t \|v_\tau\|^2 d\tau\right)\right] - (1+\gamma)\left(1 + \frac{1}{4\varepsilon}\right)\|u_0\|^4 \quad (44)$$

We choose $\varepsilon > 0$ such that $max\left\{1+4\varepsilon, 1+\frac{1}{4\varepsilon},\right\} = \frac{1+\beta}{1+\gamma}$. By assumption (8) and inequality (44) we get the estimation

$$\psi\psi'' - (1+\gamma)(\psi')^2 \geq -2\lambda\psi\psi' + ((p+m)E(0) - D_3)C_0 - (1+\gamma)^2\|u_0\|^4.$$

The lemma can be applied if $C_0 = \frac{(1+\gamma)^2}{(p+m)E(0)-D_3}\|u_0\|^4$. (45)

3 Conclusion

We get the relation $\psi\psi'' - (1+\gamma)(\psi')^2 \geq -2\lambda\psi\psi'$, with $M_1 = \lambda$, $M_2 = 0$, $\gamma_1 = 0$, $\gamma_2 = -2\lambda$. The conditions of lemma, positivity of $\psi(0) > 0$and $\psi'(0) > -\gamma_2\gamma^{-1}\psi(0)$, are satisfied by the constant (45) and the assumption (11) respectively. Thus solutions to the inverse problem for nonlinear parabolic equation (1)-(4) blow up as

$$t \to t_1 \leq \frac{1}{2\lambda} ln \frac{\gamma((p+m)E(0)-D_3)}{\gamma((p+m)E(0)-D_3)-2\lambda(1+\gamma)^2\|u_0\|^2}.$$

As a result, we find conditions on data quaranteeing global nonexistence of solution to an inverse source problem for a class of nonlinear parabolic equations.

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