Modelling of Pehlivan-Uyaroglu_2010 Chaotic System via Feed Forward Neural Network and Recurrent Neural Networks

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Abstract

In recent years, Artificial Neural Networks (ANN) have been used widely in the areas of prediction, classification, control, optimization, analysis of complex problems, and modelling of the non-linear systems. In this paper, the modelling of Pehlivan-Uyaroglu_2010 Chaotic System (PUCS) [1], which is presented by Pehlivan and Uyaroglu in 2010, is realized using Feed-Forward Neural Network (FFNN) and Recurrent Neural Networks (RNN). In the structures of each performed network models, there are 3 inputs and 3 outputs for each input and output layers respectively. Inputs describe the state variables of PUCS and outputs describe both the outputs of PUCS and the iterative version of these inputs. The numerical solution of the system's state equation is performed using Runge Kutta 5 Butcher (RK5) algorithm which is one of the differential equation solving methods. As a consequence, data which are necessary for the training and testing of FFNN and RNN, are obtained. FFNN and RNN (3-7-3) are created using MATLAB and modelled using Logistic Sigmoid (LogSig) transfer function in hidden layers. As a result of the performed modellings, the dynamics of the system and FFNN-RNN are well overlapped with the precision of 10^{-9} degree and these results are shown graphically. This study shows how to model any chaotic system and how to use transfer functions with respect to different ANN structures. In future studies, the chaotic system such as PUCS could be realized using the presented ANN models on FPGA.

Key words: Chaos, Chaotic Systems, Multi Layer Perceptron, Feed Forward Neural Network, Recurrent Neural Network

1. Introduction

1. 1.Chaotic Systems

Chaotic systems attract great numbers of researcher's interest for many years. Sensitivity to initial conditions and system parameters are the most important properties of them. In order to forecast the long term behavior of such systems, the initial conditions and system parameters must be identified precisely [2].

Pehlivan and Uyaroglu designed a new chaotic system (Pehlivan-Uyaroglu_2010 Chaotic System, PUCS) in 2010 [1]. The system is described by the following nonlinear equations in (1) and initial conditions in (2):

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$$dx / dt = y - x$$
$$dy / dt = a.y - x.z \quad (1)$$
$$dz / dt = x.y - a$$

$$x_0 = 0.001,\ y_0 = 0.001,\ z_0 = 0.001 \quad (2)$$

Heun, Euler, Runge-Kutta 4 and Runge-Kutta 5 Butcher (RK5) are among the solution methods of nonlinear equations [3]. Since RK5 produces more sensitive solutions, RK5 is chosen for this study.

The system is simulated with the system parameter of a=0.5 [1]. The system exhibits chaotic behaviour. Its attractor properties are verified by phase plane analysis.The Equation 1 has been evaluated using RK5 algorithm and the phase portraits of PUCS generated by RK5 with respect to initial conditions, namely x-y, x-z, y-z are presented in Figure 1.

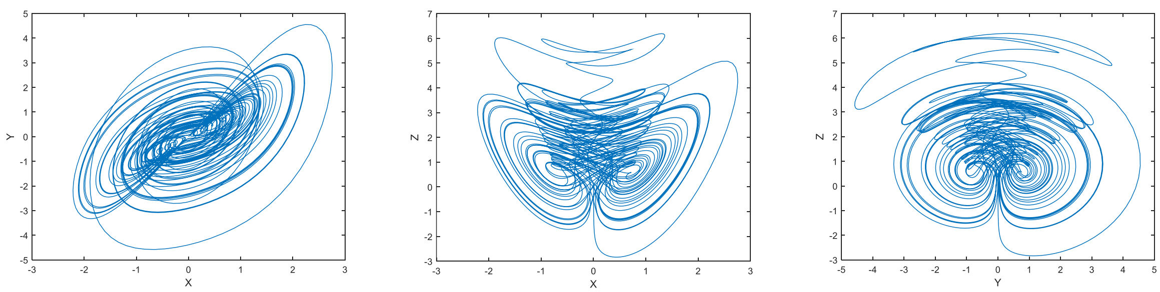


Figure 1. The phase portraits of PUCS according to initial conditions.

1. 2. System Modelling

Recently, system modelling has been used in many disciplines for several purposes. It presents many research areas including study of complex systems, modelling of nonlinear systems and study of parameters that are difficult to measure. Depending upon the studies related to Artificial Intelligence (AI), Smart Modelling concept has been developed and applied in many areas. When there is limited knowledge about the system, modelling with Smart structures may be chosen and executed according to input-output relations [4]. The main objective of this study is to model the proposed PUCS with respect to initial conditions and system parameters using ANN. In addition to modelling, the performance comparison of FFNN and RNN is among other objectives.

1. 3. Artificial Neural Network

Artificial Neural Networks (ANN) have become an important modelling tool frequently used in applications and analyzing the complex problems in different disciplines [5,6]. They are known for their adaptive capabilities which allow them to change their weights based on the available information and capture the non-linear dynamic behavior which frequently occurs in real-world systems. They contain simple processing elements similar to biological neurons in human brain [7]. There are mainly two types of Neural networks, namely Recurrent Neural Networks (RNN) and Feed Forward Neural Networks (FFNN). RNN have not only feed forward connections but also feedback (recurrent) connections as shown in Figure 2. This situation gives them memory ability [8,9,10].

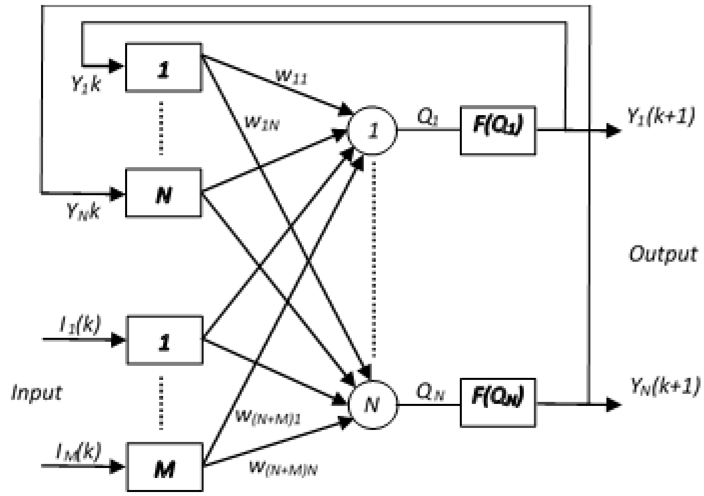


Figure 2. Structure of Recurrent Neural Network [3]

Apart from the RNN, FFNN have only feed forward connections. They have no feedback elements. They contain no delays. Thus, the output is simply calculated from the input through feed forward connections [8]. Figure 3 presents the structure of FFNN where b_1^1 denotes the bias value related to each neuron and w_1^1 denotes the weight value related to each connection.

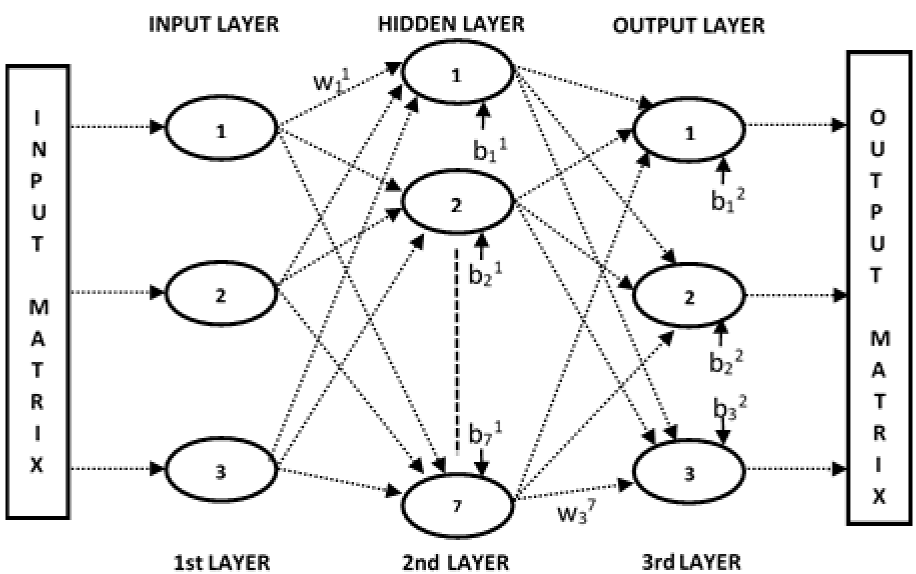


Figure 3. Structure of Feed Forward Neural Network

2. Materials and Method

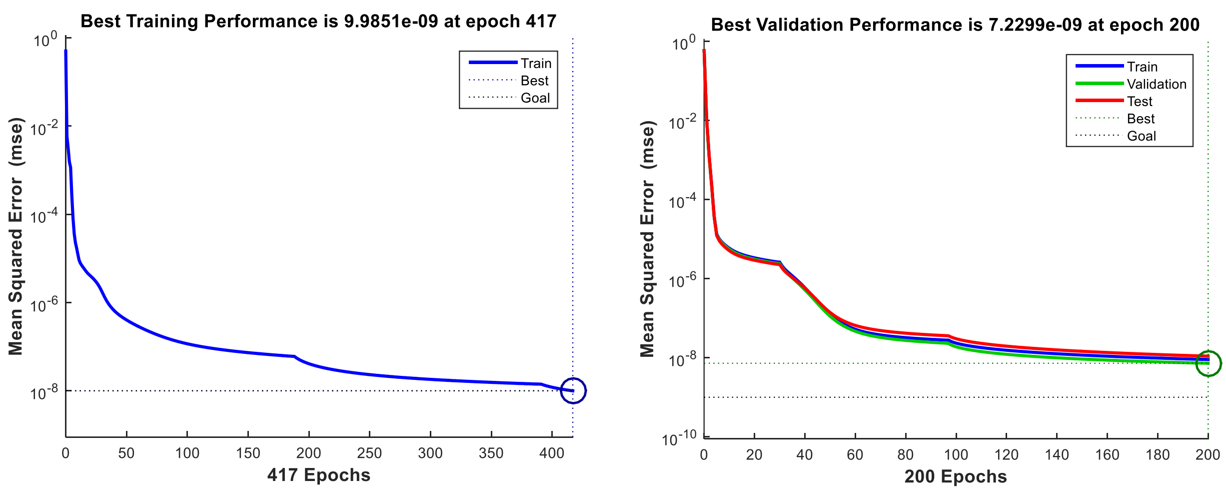
In this simulation work, PUCS, one of the chaotic system that presented by Pehlivan and Uyaroglu to the literature in 2010, is used for modelling. Through the modelling process, FFNN and RNN are created in MATLAB 2014b. Both of FFNN and RNN are composed of 3x7x3 structure. As shown in Table 1, they both have Log-Sig transfer function in hidden layer, Purelin transfer function in Output layer. Inputs and outputs consist 10000 samples generated from RK5 algorithm. Inputs and outputs represent state variables of PUCS and iterative version of these inputs respectively. Table 1 presents the initial conditions and other parameters used for calculation and design.

Table 1. Parameters of FFNN and RNN

	Model Structure	
	FFNN	**RNN**
Initial Conditions	x_0=0.001, y_0=0.001, z_0=0	x_0=0.001, y_0=0.001, z_0=0
Number of Layers	2	2
Number of Neurons per Layer	Input: 3 Hidden:7 Output:3	Input: 3 Hidden:7 Output:3
Initial weights and biases	Random	Random
Transfer Function	Log-Sigmoid Linear	Log-Sigmoid Linear
	Training Parameters	
Learning Rule	Levenberg-Margquard Back Propogation	Levenberg-Margquard Back Propogation
Mean Square Error (MSE)	9.99e-09	7.23e-09

3. Results

In this presented work, the training of FFNN and RNN have been performed according to parameters of Table 1. The training performances of FFNN and RNN are given in Figure 4. It is possible to say that RNN model is much faster and much sensitive than FFNN.



(a) **(b)**

Figure 4. **(a)** Training performance of FFNN **(b)** Training performance of RNN

After completing the learning phase, FFNN and RNN models were analyzed in Figure 5. Both of network structures have shown good modelling characteristics. Both of RK5 and FFNN-RNN are well overlapped with the precision of 10^{-9} degree. In FFNN, MSE reaches 10^{-9} degree at epoch 417. On the other hand, in RNN, MSE reaches 10^{-9} degree at epoch 200.

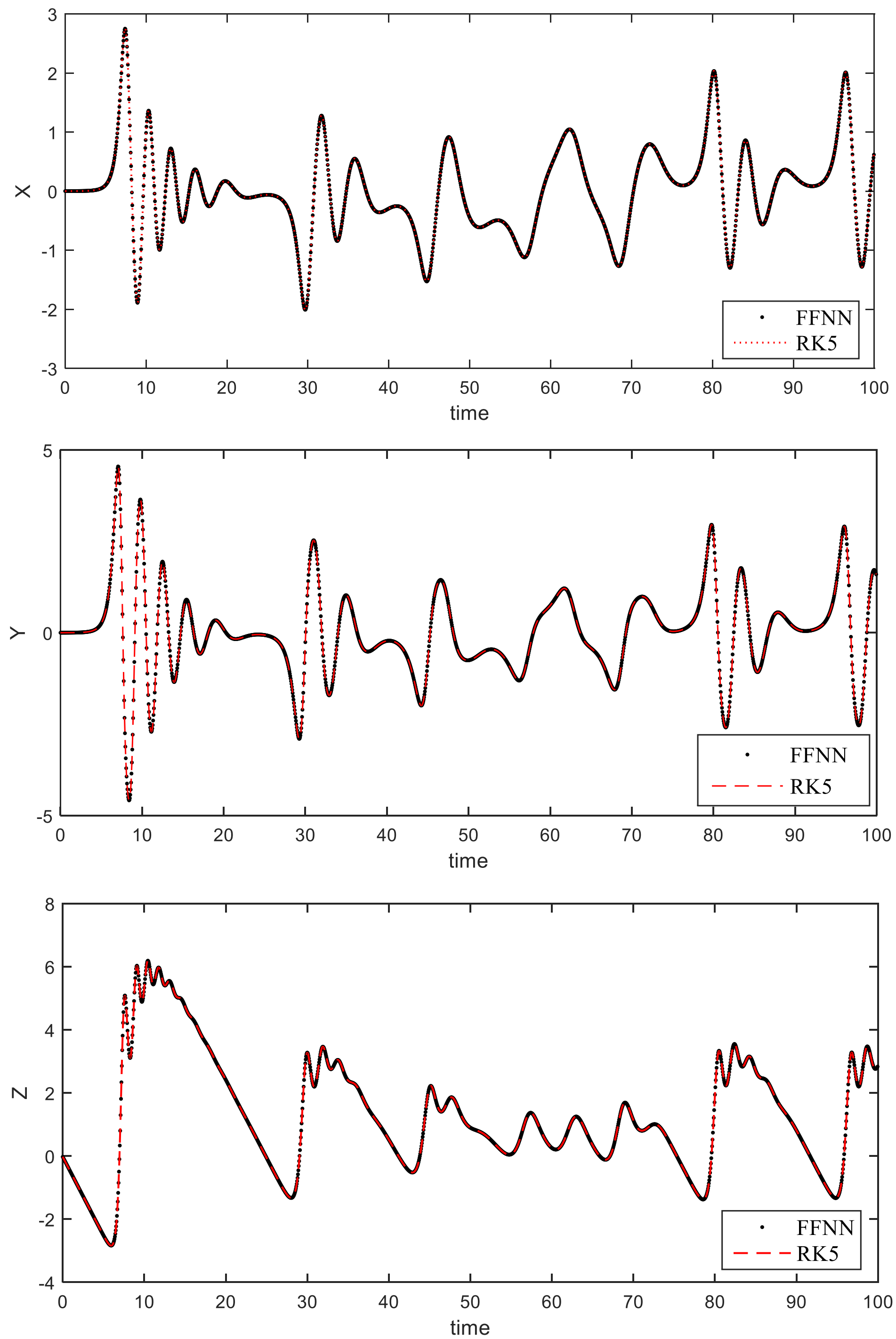


Figure 5. The change of state variables with respect to time in FFNN and RK5.

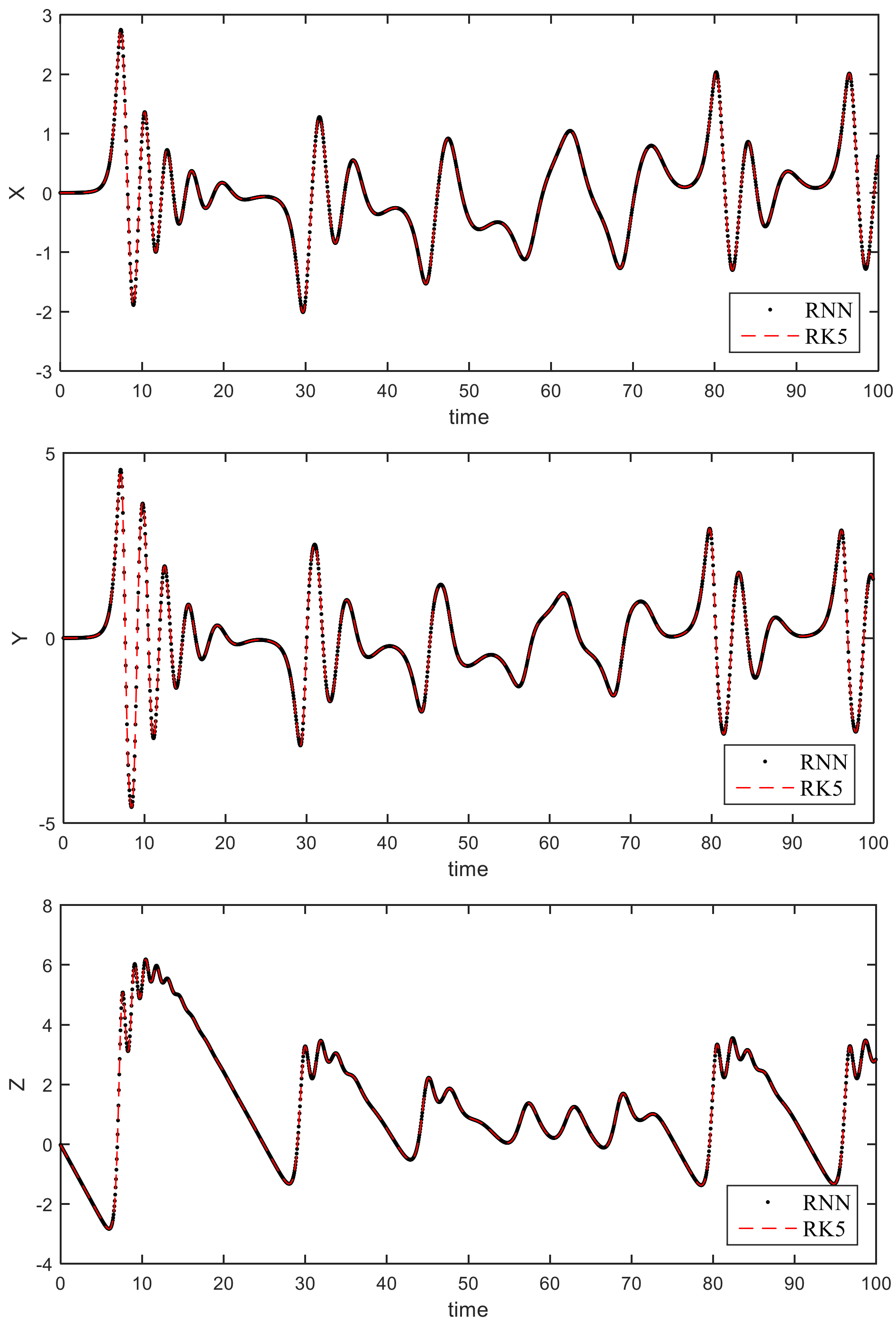


Figure 6. The change of state variables with respect to time in RNN and RK5.

4. Discussion and Conclusions

As a result of the performed modellings of the PUCS, the dynamics of the system and FFNN-RNN are well overlapped with the precision of 10^{-9} degree and these results are shown as graphically.

Although analyzed results look like similar in Figure 5 and Figure 6, it can be concluded that the best modelling performance was achieved with RNN architecture. Since the precision of the results is significantly important in chaotic systems, it is possible to model the system with better precision and less training time using RNN.

This study also shows how to model any chaotic system and how to use transfer functions with respect to different ANN structures. In future studies, any chaotic system such as PUCS could be realized using the presented ANN models on FPGA platform.

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