CFD Analysis of Forced Convection Flow and Heat Transfer in Semi-Circular Cross-Sectioned Micro-Channel

*[1]Hüseyin Kaya, [2]Kamil Arslan
[1]Bartın University, Mechanical Engineering Department, Bartın, Turkey
[2]Karabük University, Mechanical Engineering Department, Karabük, Turkey

Abstract

In this study, forced convection flow and heat transfer characteristics in semi-circular cross-sectioned micro-channel were studied numerically. Water, ethylene glycol (EG) and engine oil were used as working fluid. The three-dimensional numerical study was carried out under steady-state laminar flow. Flow was hydrodynamically and thermally developing (simultaneously developing flow) under uniform surface heat flux boundary condition on the bottom wall of micro-channel. Convective heat transfer coefficient and pressure drop were analyzed using different fluids (water, EG and engine oil). The average Nusselt number and average Darcy friction factor were determined using the numerical results. Also, the results were given with empirical correlations as: $Nu = aRe^{b}Pr^{c}$ and $f = dRe^{e}$. The highest heat transfer rate and pressure drop are obtained for engine oil and water, respectively.

Key words: CFD, semi-circular cross-sectioned, micro-channel, laminar flow, forced convection,

1. Introduction

Micro and mini-channels applications plays very important roles for effective cooling in microelectronic devices, and also used for applications such as micro-biochips, micro-reactors, refrigeration systems and micro-fuel cells. Through these micro-fluidic systems, micro-channels have been identified to be one of the fundamental elements to transport fluid within a tiny area. Circular micro-channels are considerable for increasing the length of the path of the fluid flow, increment mixing efficiency, and improving heat transfer performance within a bounded and uniform space. It is clearly understood in the past numerical and experimental studies that the fluid flow characteristics in micro-channels usually different from the macro-channels. As an illustration, the friction factor of laminar flow in micro-channels was higher than macro-channels [1]. It is important that estimating and developing correlations for pressure drop and forced convective heat transfer in micro-channels under developing flow conditions. Semi-circular cross sectioned micro-channels heat transfer and fluid flow are more complicated than circular cross-sectioned ones [2].

Several studies have been made for semi-circular cross-sectioned channels at macro level. An extensive review of forced convection flow in circular and non-circular duct cross-sections were presented by Shah and London [3], Kakaç *et al.* [2], and Kakaç and Liu [4]. An experimental investigation made by Berbish *et al.* for turbulent forced convection heat transfer and pressure

*Corresponding author: Hüseyin Kaya, Address: Faculty of Engineering, Department of Mechanical Egineering Bartın University, 74000, Bartın, TURKEY. E-mail address: hkaya@bartin.edu.tr, Phone: +903782949179

drop characteristics of air flow inside a horizontal semi-circular cross-sectioned macro-channel. The variations of surface and mean temperatures, local heat transfer coefficient, local Nusselt number, and local friction factor with the axial dimensionless distance throughout the duct were presented in this investigation. Empirical correlations were determined for the average Nusselt number and average Darcy friction factor as a function of the Reynolds number [5]. Manglik and Bergles [6] researched numerically constant property, laminar flow heat transfer in a semi-circular tube with a uniform wall temperature. A numerical investigation for steady state turbulent flow in a smooth semi-circular cross-sectioned duct at macro level was carried out by Arslan [7]. In this study flow was hydrodynamically and thermally developing under uniform surface heat flux with uniform peripheral wall heat flux. Local heat transfer coefficient and local Darcy friction factor as a function of dimensionless position along the duct obtained with numerical solutions. Hakan and Öztop [8] carried out an analytical investigation of entropy generation for laminar flow along the semi-circular cross-sectioned ducts depending on the constant wall heat flux boundary conditions. The effects of cross-sectional area and the wall heat flux on entropy generation were investigated. Geyer *et al.* [9] numerically analyzed fully developed laminar flow and heat transfer in periodic semi-circular cross-sectioned trapezoidal channels. Semi-circular cross-sectioned periodic trapezoidal channel flow has higher rates of heat transfer with relatively small pressure loss compared to flow in straight pipe.

At the micro level for laminar flow in semi-circular cross-sectioned channels, there has been less study than macro-level. Languri and Hooman [10] numerically analyzed a slip flow in a micro-channel under forced convection. Velocity slip and temperature jump boundary conditions were applied at the uniformly heated walls. Hooman [11] developed a superposition approach to investigate forced convection for arbitrary cross-sections. The flow was assumed slip-flow regime. The numerical results for parallel plate, circular and rectangular cross-sections were obtained.

Energy and momentum analysis in semi-circular cross-sectioned channels are very complicated. For designing of thermal equipment, fundamental knowledge on the flow and heat transfer of the laminar forced convection in semi-circular cross-sectioned channel is needed. On the other hand, semi-circular micro-channel geometry under laminar flow condition has not been sufficiently studied in detail.

In this study, three-dimensional flow was examined for horizontal straight semi-circular cross-sectioned micro-channel under hydrodynamically and thermally developing flow condition. The study was conducted in the laminar flow region. Water (Pr=5.83), ethylene glycol (Pr=151) and engine oil (Pr=6400) were used as the heat transfer medium. The continuity, momentum and energy equations for three-dimensional flow in the hydrodynamic and thermal entrance region of semi-circular cross-sectioned micro-channel were solved using finite volume based commercial software ANSYS Fluent 15.0. The changing of average Nusselt number and average Darcy friction factor with Reynolds number were analyzed. Applicable engineering correlations for average Nusselt number and average Darcy friction factor were also presented. It was obtained in the numerical study that; convection heat transfer in semi-circular cross-sectioned micro-channel for engine oil flow was greater than water and EG flow. On the other hand, the pressure drop of the micro-channel was highest in water flow.

2. Mathematical modeling and Computation Method

The schematic diagram illustrating the cross-section and computational territory of the semi-circular cross-sectioned micro-channel along with the coordinate system and dimensions of the flow geometry were given in Figure 1. Mathematical modeling was made for micro-channel to analyze the case. The three-dimensional Navier-Stokes and energy equations were used to identify the flow and heat transfer in the computational domain. The physical properties of fluids were taken for the bulk temperature in the micro-channel.

The continuity, momentum and energy equations in cylindrical coordinate system are given below.

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial (rv)}{\partial r}+\frac{1}{r}\frac{\partial w}{\partial \theta}=0 \tag{1}$$

Axial momentum equation:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}+\frac{w}{r}\frac{\partial u}{\partial \theta}\right)=-\frac{\partial P}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right] \tag{2}$$

Radial momentum equation:

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}+\frac{w}{r}\frac{\partial v}{\partial \theta}-\frac{w^2}{r}\right)=-\frac{\partial P}{\partial r}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv)\right)+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial w}{\partial \theta}\right] \tag{3}$$

Angular momentum equation:

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial r}+\frac{w}{r}\frac{\partial w}{\partial \theta}-\frac{vw}{r}\right)=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rw)\right)+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v}{\partial \theta}\right] \tag{4}$$

Energy equation:

$$\begin{aligned}\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}+\frac{w}{r}\frac{\partial T}{\partial \theta}\right)&=k\frac{\partial^2 T}{\partial x^2}+\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{k}{r^2}\frac{\partial^2 T}{\partial \theta^2}+2\mu\left(\frac{\partial u}{\partial x}\right)^2+2\mu\left(\frac{\partial v}{\partial r}\right)^2\\ &+\frac{2\mu}{r^2}\left(\frac{\partial w}{\partial \theta}+v\right)^2+\mu\left(\frac{\partial w}{\partial x}+\frac{1}{r}\frac{\partial u}{\partial \theta}\right)^2+\mu\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)^2+\mu\left[\frac{1}{r}\frac{\partial v}{\partial \theta}+r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right]^2\end{aligned} \tag{5}$$

The continuity, momentum and energy equations were solved using ANSYS Fluent 15.0. The fluid enters the duct with uniform velocity and temperature profile. At the inlet hydraulic diameter of the channel was used as convenient length scale. No slip boundary conditions were used on the channel walls. A uniform heat flux $q''=10\ kW/m^2$ was applied on the bottom surface of the channel, while the rest of the micro-channel was thermally isolated from the surroundings. Also, each fluid enters the micro-channel at $Ti=300\ K$ and ANSYS Fluent 15.0 pressure outlet boundary condition was performed at the outlet of the channel.

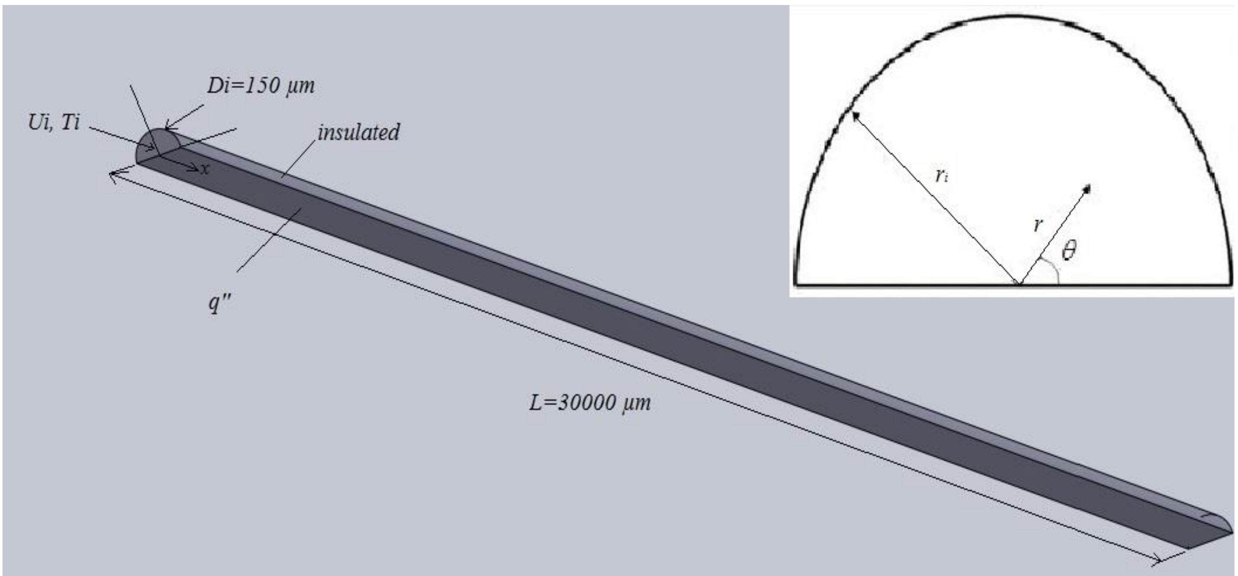


Figure 1. Schematic diagram of semi-circular cross-sectioned micro-channel with boundary condition

Average Nusselt number, average Darcy friction factor and Reynolds number were determined by:

$$\mathrm{Re} = \frac{UD_h}{\nu} \quad (6) \qquad Nu = \frac{hD_h}{k} \quad (7) \qquad f = \frac{\Delta p\left(\frac{D_h}{L}\right)}{\frac{\rho U^2}{2}} \quad (8)$$

The hydraulic diameter, D_h, is chosen as the characteristic length and obtained from the Eq. (9) for the semi-circular cross-sectioned micro-channel [7]. Average heat transfer coefficient is also determined [12] from Eq. (10).

$$D_h = \frac{\pi D_i}{\pi + 2} \quad (9) \qquad h = \frac{q''}{T_w - T_b} \quad (10)$$

Additionally, all fluid properties in the micro-channel is taken at the bulk temperature of fluid $T_b = (T_{bi} + T_{bo})/2$ [13].

In this study, a general finite-volume based commercial CFD software ANSYS Fluent 15.0 was used to perform the numerical investigation. The code provides the mesh flexibility by structured and unstructured meshes. Computations were carried out under laminar flow conditions. The energy equation was solved for negligible radiation effects. The Navier-Stokes equations were solved numerically with transport equations for laminar flow. In this study, tetrahedron cells were generated with a fine mesh near the plate walls. In order to obtain fine mesh distribution boundary layer mesh was used adjacent the surfaces of the micro-channel. The non-uniform grid distribution was performed in the plane perpendicular to the main flow direction (Figure 2). Symmetry applied to the geometry to simplify the calculations. The number of grid points or control volumes close to the each wall was increased to augment the resolution and accuracy.

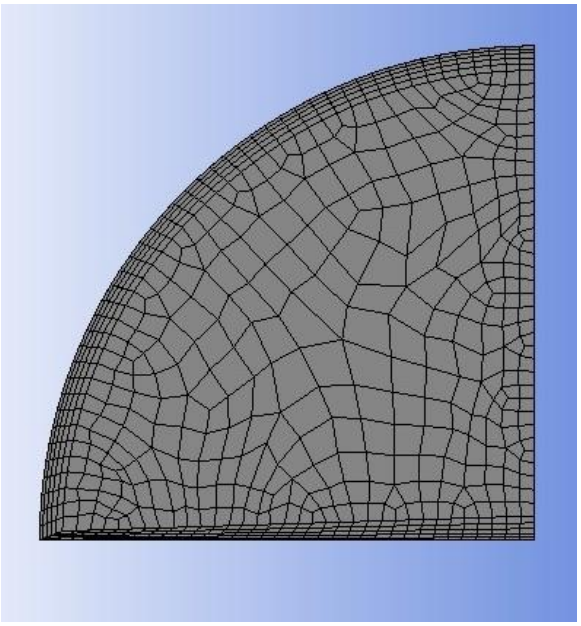


Figure 2. Mesh distribution on y-z plane

The mesh independence study was carried out by clarifying the grid size until the variation in both average Nusselt number and average Darcy friction factor were less than about 0.3% and 1.2%, respectively. In order to satisfy certainty of results presented, a grid independence study was performed for EG flow in micro-channel using ten different grid sizes changing from $1.36\cdot10^5$ to $1.74\cdot10^6$ for Re = 1000 to study the effects of grid size. It is recognized that further clarification of grids from $9.07\cdot10^5$ to $1.74\cdot10^6$ did not have remarkable effect on the results with regards to average Nusselt number and average Darcy friction factor as illustrated in Figure 3. The grid size of $9.07\cdot10^5$ points was used for all of the calculations depending on this mesh independence process. The grid size was also used for Reynolds number from 100 to 1000.

Each equation for continuity, momentum and energy has been iterated until the residual drops below $1\cdot10^{-6}$ for achieving convergence of the solution. There is no converging solution problem for iterations of each working fluid.

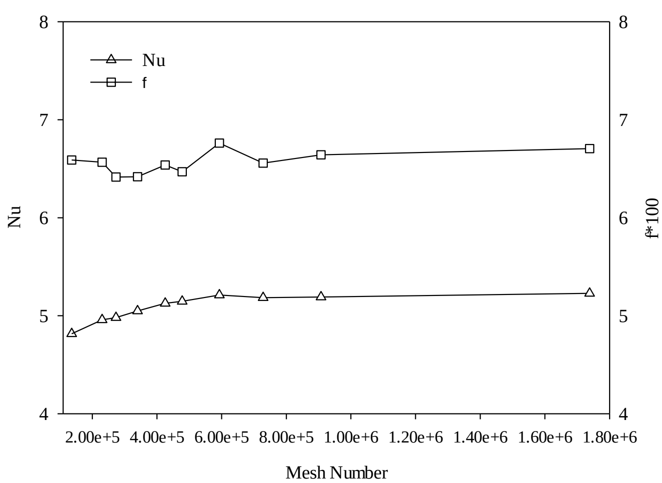


Figure 3. Changing of average Nusselt number and average Darcy friction factor with mesh number for EG flow

3. Results and Discussion

In this study, the convective heat transfer and fluid friction in an air-cooled semi-circular cross-sectioned micro-channel under uniform bottom surface heat flux with isolated peripheral wall was numerically analyzed. The analysis was conducted under hydrodynamically and thermally developing laminar flow condition. The results were given with average Nusselt number and average Darcy friction factor. The average Nusselt numbers were calculated after the determination of temperature fields in each fluid. Additionally, the values were estimated for average Darcy friction factors after finding out of pressure drop in the channel.

In order to test the numerical code, results for water flow in micro-channel were compared with the experimental results obtained from the literature [14]. It was seen in the Figure 4 that the current numerical study was harmonious with the experimental data. The maximum deviation between the numerical and experimental results was obtained almost ±4%.

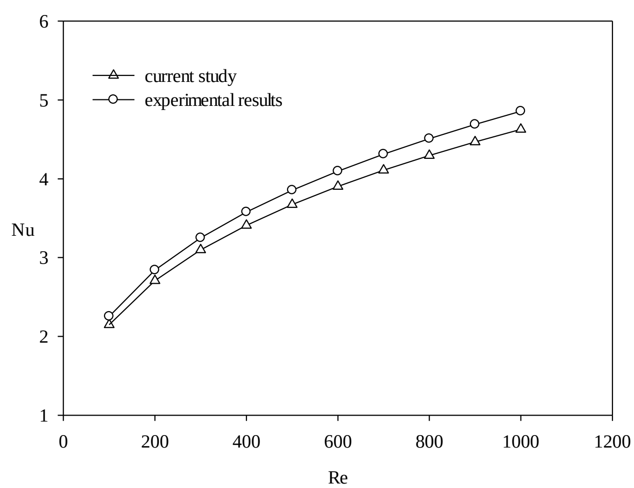
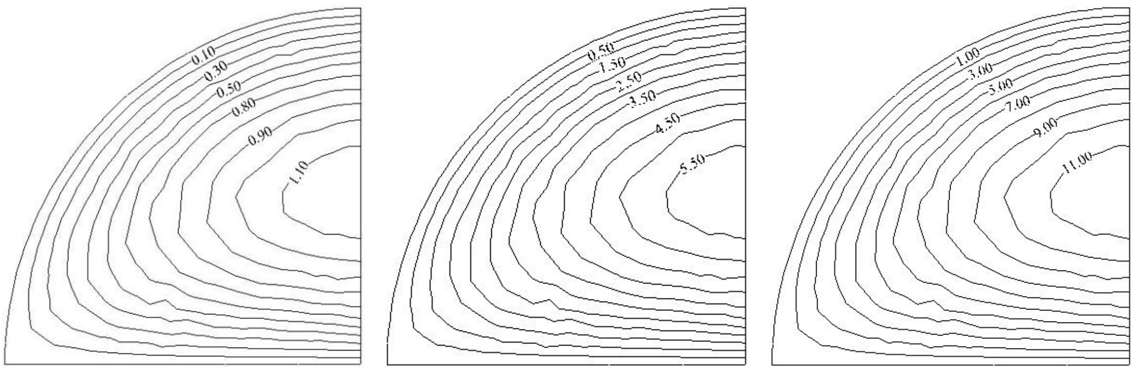


Figure 4. Comparison between the numerical investigation and literature

The numerical results under steady-state conditions were shown in Figure 5-10. This study was carried out in order to emphasize the effect of channel geometry and wall boundary conditions on thermal performance of semi-circular channel. The flow velocity and temperature distributions were presented as well.

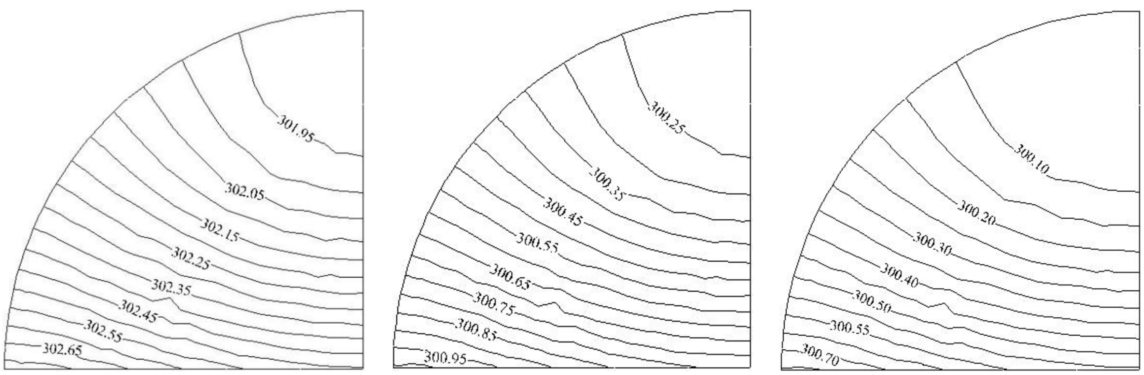
Velocity and temperature distribution of water flow at the outlet of the duct for different Reynolds number is shown in Figure 5 and 6, respectively. It was seen that the velocity and temperature values changed with changing Reynolds number. Maximum velocity was obtained at the center of the channel. At the same time, the temperature of the fluid decreases from bottom surface to top surface of the micro-channel.

Figure 7 and 8 show the velocity and temperature distributions at the outlet of the duct for different fluids at Re=1000, respectively. It is obtained in these figures that velocity and temperature profiles change with working fluid.



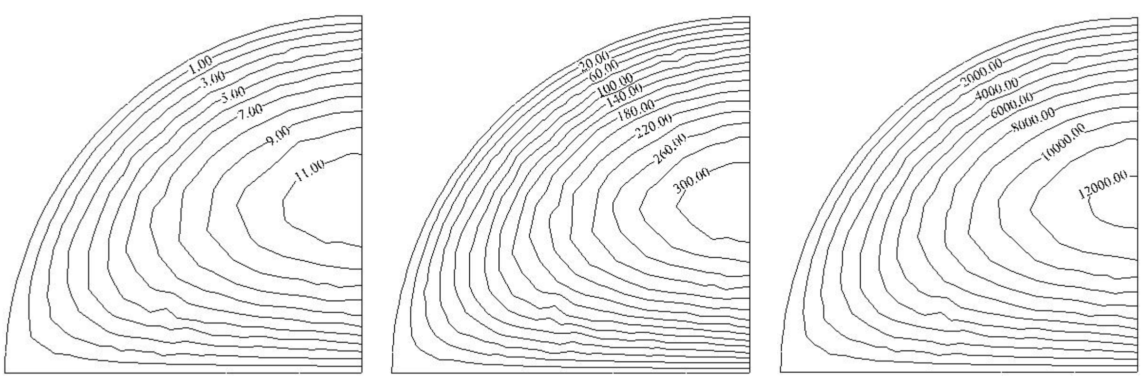
(a) **(b)** **(c)**

Figure 5. Velocity distributions at the outlet of the micro-channel for water flow; **(a)** Re=100, **(b)** Re=500, **(c)** Re=1000.



(a) **(b)** **(c)**

Figure 6. Temperature distributions at the outlet of the micro-channel for water flow; **(a)** Re=100, **(b)** Re=500, **(c)** Re=1000.



(a) **(b)** **(c)**

Figure 7. Velocity distributions at the outlet of the micro-channel for different fluids at Re=1000; **(a)** Water, **(b)** EG, **(c)** Engine Oil.

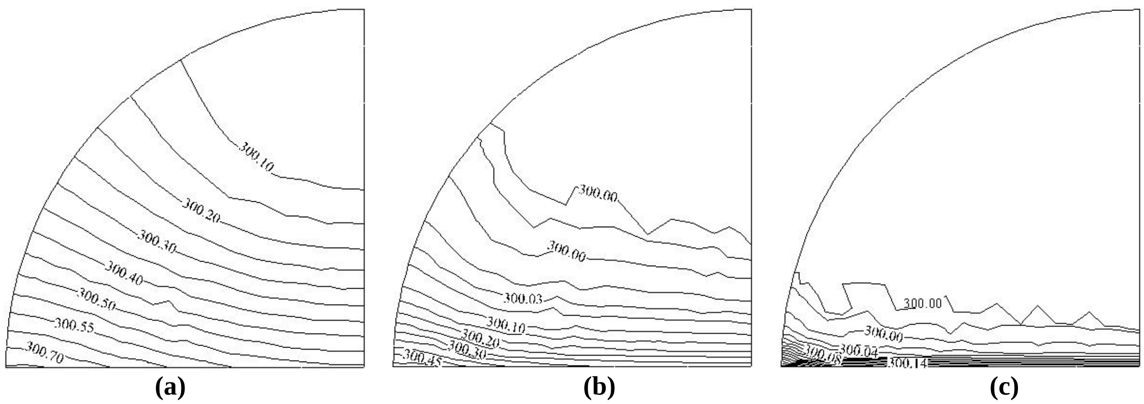


Figure 8. Temperature distributions at the outlet of the micro-channel for different fluids at Re=1000; **(a)** Water, **(b)** EG, **(c)** Engine Oil.

Figure 9 and 10 represent the numerical results for each working fluid flow in semi-circular cross-sectioned micro-channel. It is obtained in the figures that, average Nusselt number increases with increasing Reynolds number, while Darcy friction factor decreases as expected. The engine oil has the highest value of Nusselt number at the same boundary condition. It represents that, the convection heat transfer rate of engine oil is more effective than the other fluids. Also, the friction factor values for water greater than others. The reasons of this circumstance, greater viscosity and smaller Prandtl number for water than engine oil and ethylene glycol. Besides, it is seen form in Figure 10, EG and engine oil almost the same friction values at the same Reynolds numbers.

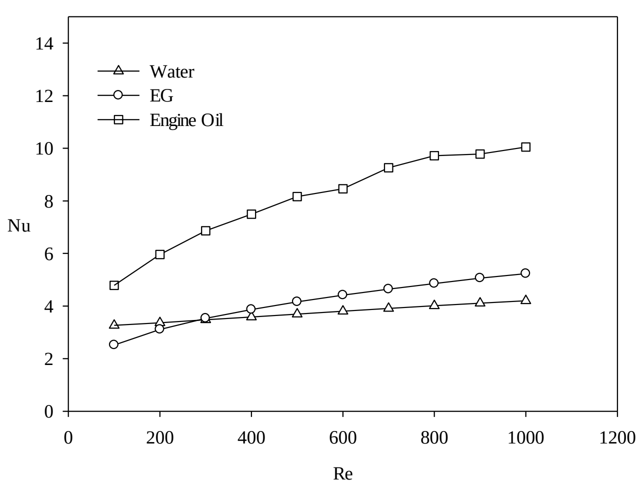


Figure 9. Variation of Nusselt number with Reynolds number for different working fluid

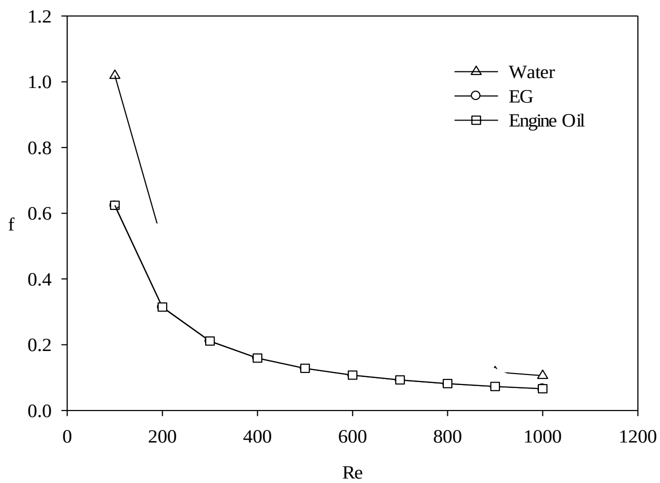


Figure 10. Variation of Darcy friction factor with Reynolds number for different working fluid

The applicable correlations of Nusselt number and Darcy friction factor were obtained from numerical investigation of flow in a semi-circular cross-sectioned micro-channel of each working fluid. The empirical correlations for Nusselt number and the Darcy friction factor are given as:

$$Nu = 0.3836\,\mathrm{Re}^{0.30}\,\mathrm{Pr}^{0.13} \tag{11}$$

$$f = \frac{70.53}{\mathrm{Re}} \tag{12}$$

4. Conclusions

The numerical investigation was carried out of heat transfer and friction factor for hydrodynamically and thermally developing three-dimensional steady laminar flow in a semi-circular cross-sectioned micro-channel. The analysis was made with different Reynolds number ranging from 10^2 to 10^3 of each working fluid. Numerical results for laminar case of each working fluid are compared with each other and the literature. Numerical computations results were given in terms of average Nusselt numbers and average Darcy friction factors. There was an increment of average Nusselt number with increasing Reynolds number. On the contrary, average Darcy friction factor decreases with increasing Reynolds number. Velocity and temperature distribution in micro-channel were also presented graphically for different Reynolds number and working fluid. According to numerical calculations of 3-D governing equations in the hydrodynamic and thermal entrance region of laminar flow, new engineering correlations were determined for the average Nusselt number and average Darcy friction factor. It was obtained in this study that, the friction and heat transfer coefficients depend on Reynolds number and working fluid for laminar fluid flow in semi-circular cross-sectioned micro-channel.

References

[1] Guobing Z., Shi-Chune Y., "Effect of surface roughness on laminar liquid flow in micro-channels" Applied Thermal Engineering, 2011; 31, 228-234.
[2] Kakac, S., *et al.*, Handbook of Single-Phase Convective Heat Transfer, John Wiley and Sons, 1987, ch 3, p. 4.
[3] Shah, R. K., London, A. L., "Laminar Flow Forced Convection in Ducts" Academic Press Inc., New York, USA, 1978, 256-259.
[4] Kakac, S., Liu, H., Heat Exchanger Selection, Rating and Thermal Design, 2nd ed. USA, 2002, p. 81-127.
[5] Berbish, N. S., *et al.*, "Heat transfer and Friction Factor of Turbulent Flow through a Horizontal Semi-Circular Duct" Heat and Mass Transfer, 2011; 47, 377-384.
[6] Manglik, R. M., Bergles, A. E., "Laminar Flow Heat Transfer in a Semi-Circular Duct with Uniform Wall Temperature, International Journal of Heat and Mass Transfer, 1988; 80, 625-636.
[7] Arslan K., "Three Dimensional Numerical Investigation of Turbulent Flow and Heat Transfer Inside a Horizontal Semi-Circular Cross-Sectioned Duct", Thermal Science, 2014; 18, 1145-1158.
[8] Hakan, F., Oztop, H. F., "Effective Parameters on Second Law Analysis for Semi-Circular Ducts in Laminar Flow and Constant Wall Heat Flux", International Communications of Heat and Mass Transfer, 2005; 32, 266-274.
[9] Geyer, P. E., *et al.* "Laminar Flow and Heat Transfer in a Periodic Trapezoidal Channel with Semi-Circular Cross-Section", Interational Journal of Heat and Mass Transfer, 2007; 50, 3471-3480.
[10] Languri E. M., Hooman K., "Slip Flow Convection in a Microchannel with Semi-Circular Cross-Section", International Communications in Heat and Mass Transfer, 2011; 38, 138-143.
[11] Hooman K., "A Superposition Approach to Study Slip-Flow Forced Convection in Straight Microchannels of Uniform but Arbitrary Cross-Section", 2008; 51, 3753-3762.
[12] Cengel, Y. A., Heat Transfer a Practical Approach, McGraw-Hill New York, USA, 1988 p. 372-380.
[13] Incropera, F. P., DeWitt, D. P., Fundamentals of Heat and Mass Transfer, 5th ed. John Wiley and Sons, New York, USA, 2002.
[14] E.N. Sieder, G.E. Tate, Heat transfer and pressure drop of liquids in tubes, Ind. Eng. Chem., 1936; 28 (12), 1429–1435.