Design Algorithm of Air Core Multilayer Tapped Reactor

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Abstract

In this paper, the design of roundwire dry type air core tapped reactor is considered. An design model is built based on the balance of Kirchhoff voltage law equations appied on all layers of the tapped reactor. The algorithm starts using a selected inner diameter of the tapped reactor. The algorithm calculates both power loss and weight of the tapped reactor for the selected inner diameter value. As a result of the algorithm, the design values are found weight, power loss, height, number of turns of coils of the tapped reactor. The algorithm has a lot of mathematical calculations on magnetic field theory. One of them is mutual inductance calculations of the parallel connected layers. The other one ise self inductance calculations of the all layers of the all packages. The all mathematical formulations are based on the Lorenz, Maxwell's etc. formulas for self inductance calculations of the layers and complete elliptic integral of the third kind for mutual inductance calculations of the parallel connected layers of the tapped reactor.

Key words: Tapped reactor, multilayer, Lorenz, Maxwell.

1. Introduction

Air core tapped reactor is used for the current limit, voltage regulator and reactive power compensation in power system [1]. However, it is actually difficult to design the reactor. It has very important design parameters such as the number of layers, the turn number of windings, the inner diameter, the number of packages, the wire diameter of layers. There are relationship among the parameters, for example, the turn number of spins and the height of layers. Another problem will be terrible circulating current among layers, if the reactor is improperly designed [2–6].

Different methods have been given in various papers to design air core tapped reactor. Which mainly include layer and package current equalization, current density equalization, package temperature rise equalization and layer resistance drop equalization [7–12]. Especially in [12], there are a lot of parameters to find their values very difficult.

2. Calculation Methods

Input parameters of producing of the air core tapped reactor are frequency, nominal current value, inductance values of all taps, material of wire of layers (Al or Cu), dencity of currents of the

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layers, inner diameter of the tapped reactor, number of package of the tapped reactor, layer number of the tapped reactor, number of parallel connected layers in a package, thickness of the paper between two layers, thickness of the dog bone between two packages, diameters of layers from the outer layer to the inner layer wire with izolation or not respectively, (if necessary) short circuit current for the tapped reactor, (if necessary) beginning and the last temperature of the layer, (if necessary) short circuit time of the tapped reactor. Output parameters of producing of the air core tapped reactor (for all taps) are actual nominal current value of the tapped reactor, the angle of current of the tapped reactor, dc resistance of the tapped reactor both 25 C^o and 75 C^o, reactive power of the tapped reactor, quality factor of the tapped reactor, number of spirs of all layers of all packages, total length of the wire of the all layers, total power loss of the tapped reactor, total weight of the tapped reactor, height of the tapped reactor, total weight of the wire of the tapped reactor, (actual) inductance of the tapped reactor for all taps. Cross section of an air core 4 tapped reactor (horizontal position) has been shown in fig. 1. In this fig., the reactor has 4 packages and a package has only one layer. In fig. 2, a real air core tapped (5 taps)

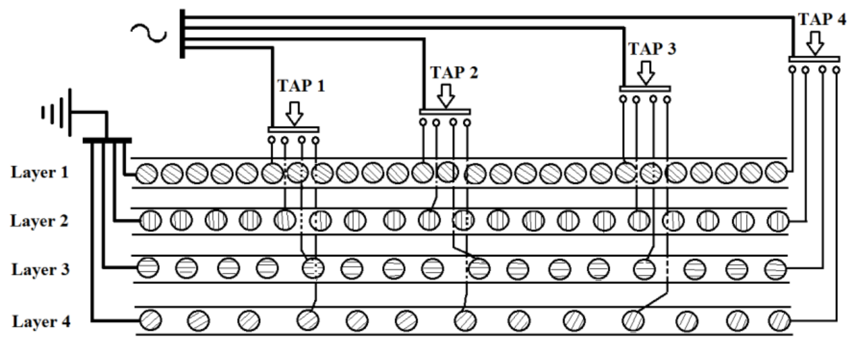

Figure 1. Cross section of an air core 4 tapped reactor (horizontal position). The reactor has 4 packages and a package has only one layer.

Figure 2. Air core tapped reactor (with 5 taps)

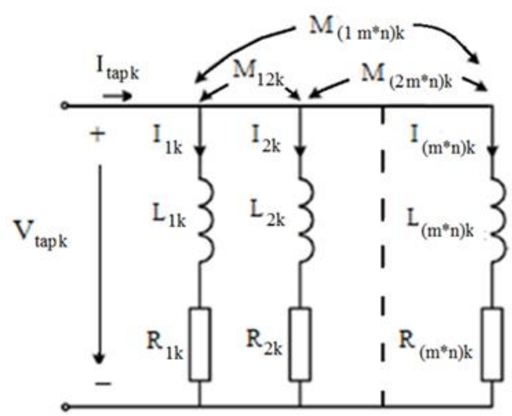

Figure 3. Circuit model of all packages of an air core tapped reactor for tap k.

reactor in a switchyard has been shown. It can be obtained circuit model shown for all layers of an air core tapped reactor for tap k in fig.3. Suppose the total number of packages is n, the total number of layers in a package is m. Number of all layers of the tapped reactor is m*n. Kirchhoff voltage equations for all layers of the tapped reactor for tap k can be written as;

$$
\begin{aligned}
&jwL_{1k}I_{1k} + jwM_{(12)k}I_{2k} + ...+ jwM_{(1j)k}I_{jk} + ...+ jwM_{(1\,m*n)k}I_{(m*n)k} + R_{1k}I_{1k} = V_{tap\,k} \\
&jwM_{(21)k}I_{1k} + jwL_{2k}I_{2k} + ...+ jwM_{(2j)k}I_{jk} + ...+ jwM_{(2\,m*n)k}I_{(m*n)k} + R_{2k}I_{2k} = V_{tap\,k} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1a) \\
&jwM_{(i1)k}I_{1k} + jwM_{(i2)k}I_{2k} + ..+ jwL_{ik}I_{ik} + ..+ jwM_{(ij)k}I_{jk} + ..+ jwM_{(im*n)k}I_{(m*n)k} + R_{ik}I_{ik} = V_{tap\,k} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&jwM_{(m*n1)k}I_{1k} + jwM_{(m*n2)k}I_{2k} + ...+ jwL_{(m*n)k}I_{(m*n)k} + ..+ jwM_{(m*nj)k}I_{jk} + .. \\
&\qquad\qquad ...+ jwL_{(m*n,m*n)k}I_{(m*n)k} + R_{(m*n)k}I_{(m*n)l} = V_{tap\,k}
\end{aligned}
$$

Eq.(1a) can be written as matrix form like Eq. (1b);

$$
\begin{bmatrix}
R_{1k}+L_{1k} & M_{(12)k} & M_{(13)k} & .. & M_{(1,m*n)k} \\
M_{(2,1)k} & R_{1k}+L_{1k} & M_{(2,3)k} & .. & M_{(2,m*n)k} \\
M_{(3,1)k} & M_{(3,2)k} & R_{3k}+L_{3k} & .. & M_{(3,m*n)k} \\
.. & .. & .. & .. & .. \\
M_{(m*n,1)k} & M_{(m*n,2)k} & M_{(m*n,3)k} & .. & R_{(m*n)(m*n)k}+L_{(m*n)(m*n)k}
\end{bmatrix}
\begin{bmatrix} I_{1k} \\ I_{2k} \\ I_{3k} \\ .. \\ I_{(m*n)k} \end{bmatrix}
=
\begin{bmatrix} V_{tap\,k} \\ V_{tap\,k} \\ V_{tap\,k} \\ .. \\ V_{tap\,k} \end{bmatrix}
\qquad (1b)
$$

Using Kirchhoff's current law, we have equation for current vectors of all layers of the reactor for tap k;

$$I_{1k} + I_{2k} + ...+ I_{(m*n)k} = I_{tap\,k} \qquad (2)$$

where R_{ik} , L_{ik} and I_{ik} are the resistance, the self inductance and the (vector) current of the ith layer, respectively for tap k. $M_{(ij)k}$ is the mutual inductance between the ith layer and the jth layer of the tapped reactor for tap k. V_{tapk} and $I_{tap\,k}$ are voltage and current vector of the air core tapped reactor for tap k respectively. w is the angular frequency. Considering that the self inductance (L_{ik}) and the mutual inductance ($M_{(ij)k}$) given Eq. (1-2) can be expressed as the following subsections.

3. The calculation of the self inductances of the air core tapped reactor

Kirchhoff's formula, separates a coil in circular windings, like Maxwell's method, and joins mutual inductances between windings calculated by elliptic integrals with g(0), an approximation for the self inductance of a single loop. For the case of a solenoid, there is a simplification similar to the one described for Maxwell's method, with only *n* different mutual inductances that are calculated by Maxwell's formula [13]:

$$L = ng(0) + 2(n-1)\,g(\gamma) + 2(n-2)\,g(2\gamma) + ... + 2g((n-1)\gamma)\ ; \quad g(z) = \mu_o \frac{r}{k}\left[(2-k^2)K - 2E\right] \tag{3}$$

The variables used in Eq.(3) are,

$$k^2 = \frac{4r^2}{h^2+z^2}\ ; \quad g(0) = \mu_o r(\ln\frac{8r}{\beta} - \frac{7}{4})\ ; \quad \gamma = \frac{h}{n}\ ; \quad g(0) = \mu_o r(2(K-E) + \frac{1}{4})\ ; \quad k = \frac{r-\beta}{r} \tag{4}$$

$$p = \frac{2r}{h}\ ; \ \theta = \tan^{-1} p\ ; \ k = \sin\theta\ ; \ k' = \cos\theta\ ; \ z = \frac{\pi nc}{h}\ ; \ \beta = 0.5c \tag{5}$$

β is the wire radius. Eq.(4) and Eq.(5) are placed in Eq.(3), the new formulation of self inductance of a layer is,

$$L = \frac{\mu_o}{4\pi}\left\{\frac{8n^2 r\pi}{3}\left[\frac{K+(p^2-1)E}{k} - p^2\right] + 2\pi r\left[2n(0.25 - \ln z) + \frac{1}{3}\ln(\frac{2\pi nr}{h}) - \frac{4}{\pi^2}(\frac{E}{k} - 1)(1 + \frac{z^2}{8}) - \frac{2}{3}(\frac{K-E}{k} - \frac{kK}{2}) - \frac{k'}{2k}(1 - \frac{k'\theta}{k})\right] + h(\ln\frac{1+k'}{1-k'} + k'\ln 4)\right\} \tag{6}$$

In Eq. (6), K and E are the complete elliptic integrals of first and second kinds with modulus k respectively:

$$K = F(k,\pi/2) = F(k) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}}\ ; \qquad E = E(k,\pi/2) = E(k) = \int_0^{\pi/2}\sqrt{1-k^2\sin^2\varphi}\, d\varphi \tag{7}$$

In Eq. (3-7), r is radius of the layer, h is height of the layer and c is diameter of wire. n is number of spir of the layer. n must be integer.

4. The calculation of the mutual inductances of the air core tapped reactor

The formula for the mutual inductance between a circular ring and a current sheet solenoid is [14];

$$M_\theta = \frac{\mu_o}{4\pi}\Theta(A+a)ck\left[\frac{K-E}{k^2} + \frac{c'^2}{c^2}(K - \prod(k,c))\right]; \tag{8}$$

A is the radius of the solenoid, a the radius of the loop. The variables used in Eq.(8) are [15],

$$c = \frac{2\sqrt{Aa}}{A+a}\ ; \ x = p\theta\ ; c'^2 = 1 - c^2\ ; k = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2 + x^2}} \tag{9}$$

$$\prod(k,c) = \int_0^{\pi/2} \frac{d\varphi}{(1-c^2 \sin^2 \varphi)\sqrt{1-k^2 \sin^2 \varphi}}; \qquad (10)$$

p the height of a turn divided by 2π, θ is the final angle of the solenoid 2πn. Eq. (10) is the complete elliptic integral of the third kind. If Eq.(9) and Eq.(10) are placed in Eq.(8), the new mutual inductance formulation is [15],

$$M_\theta = \frac{\mu_o}{2} n\left\{ z(K-E) + \frac{(A-a)^2}{z}\left[(K - \prod(k,c)\right]\right\} \qquad (11)$$

The variables used in Eq.(11) are,

$$z = \sqrt{(A+a)^2 + x^2}; \quad k = \frac{2\sqrt{Aa}}{z}; \quad c = \frac{2\sqrt{Aa}}{A+a} \qquad (12)$$

Another formulation for Eq.(11) is [16],

$$M_\theta = \frac{\mu_o}{2x} n\left\{ \frac{2x\sqrt{Aa}}{k}(K-E) \pm \left|A^2 - a^2\right|\left[KE(k',\theta) - (K-E)F(k',\theta) - \frac{\pi}{2}\right]\right\} \qquad (13)$$

The variables used in Eq.(13) are,

$$k = \sqrt{\frac{4Aa}{x^2 + (A+a)^2}}; \quad k' = \sqrt{1-k^2}; \quad \theta = \sin^{-1}\sqrt{\frac{1+\frac{x^2}{(A+a)^2}}{1+\frac{x^2}{(A-a)^2}}} \qquad (14)$$

A formula for the mutual inductance between two solenoids modeled as current sheets [14];

$$M = \frac{2\pi n_1 n_2}{h_1 h_2}\left\{W(b_2 - b_1 + h_2) + W(b_2 - b_1 + h_1) - W(b_2 - b_1 + h_2 - h_1) - W(b_2 - b_1)\right\} \qquad (15)$$

In Eq (15), for tap k, b_1 is distance between base of the first solenoid and reference point, b_2 is distance between base of the second solenoid and reference point, h_1 is height of the first solenoid, h_2 is height of the second solenoid, n_1 is number of turn of the first solenoid, n_2 is number of turn of the second solenoid. If bases of both of solenoids are coincident with the reference point, it will be $b_1 = b_2 = 0$. The equations of $W(x)$ and $W'(x)$ used in Eq. (15) are;

$$W(x) = xW'(x) + \frac{8(r_1 r_2)^{3/2}}{3k}\left[K - (\frac{2}{k^2} - 1)(K-E)\right] \qquad (16)$$

$$W'(x)=\frac{2x\sqrt{r_1r_2}}{k}(K-E)\pm\left|r_1^2-r_2^2\right|\left[KE(k',\theta)-((K-E)F(k',\theta)-\frac{\pi}{2}\right] \quad (17)$$

In Eqs. (16-17), for tap k, r_1 is radius of the first solenoid, r_2 is is radius of the second solenoid. The other variables used in Eqs.(16-17) are given in the following equations (18-21) [10];

$$k=\sqrt{\frac{4r_1r_2}{x^2+(r_1+r_2)^2}};\quad k'=\sqrt{1-k^2};\quad \theta=\sin^{-1}\sqrt{\frac{1+\frac{x^2}{(r_1+r_2)^2}}{1+\frac{x^2}{(r_1-r_2)^2}}}, z=\sqrt{(r_1^2-r_2^2)+x^2} \quad (18)$$

$$W'(x)=x\left\{z(K-E)+\frac{(r_1^2-r_2^2)}{z}\left[K-\prod(k,c)\right]\right\};\qquad k=2\sqrt{\frac{r_1r_2}{z^2}}; \quad (19)$$

$$c=2\sqrt{\frac{r_1r_2}{(r_1+r_2)^2}};\qquad k=\sqrt{\frac{4r_1r_2}{x^2+(r_1+r_2)^2}}; \quad (20)$$

$$W'(x)=\frac{2x\sqrt{r_1r_2}}{k}(K-E)\pm\left|r_1^2-r_2^2\right|\left[\frac{\pi}{2}\left[\Lambda_o(k,\theta)-1\right]\right]\quad;\quad \theta=\sin^{-1}\sqrt{\frac{1+\frac{x^2}{(r_1+r_2)^2}}{1+\frac{x^2}{(r_1-r_2)^2}}} \quad (21)$$

5. Short circuit test of the reactor for minimum tap k

Assuming that F_1 is the initial temperature (C^o) of the reactor, F_2 the last temperature (C^o) of the reactor and F_3 is short circuit time (sn), the current density of the tapped reactor (for short circuit condition) can be calculated for aluminium (for minimum tap k);

$$F_4=\sqrt{(\frac{21800}{F_3})\log(\frac{235+F_2}{235+F_1})}\qquad (A/mm^2) \quad (22)$$

The current density of the tapped reactor for copper (for minimum tap k);

$$F_4=\sqrt{(\frac{49804}{F_3})\log(\frac{235+F_2}{235+F_1})}\qquad (A/mm^2) \quad (23)$$

The total layers cross section of the the tapped reactor (for short circuit calculations) must be greater than (for minimum tap k);

$$S = \frac{F_5}{F_4} \quad (mm^2) \qquad (24)$$

F_5 is short circuit current (A) for the tapped reactor in Eq.(24).

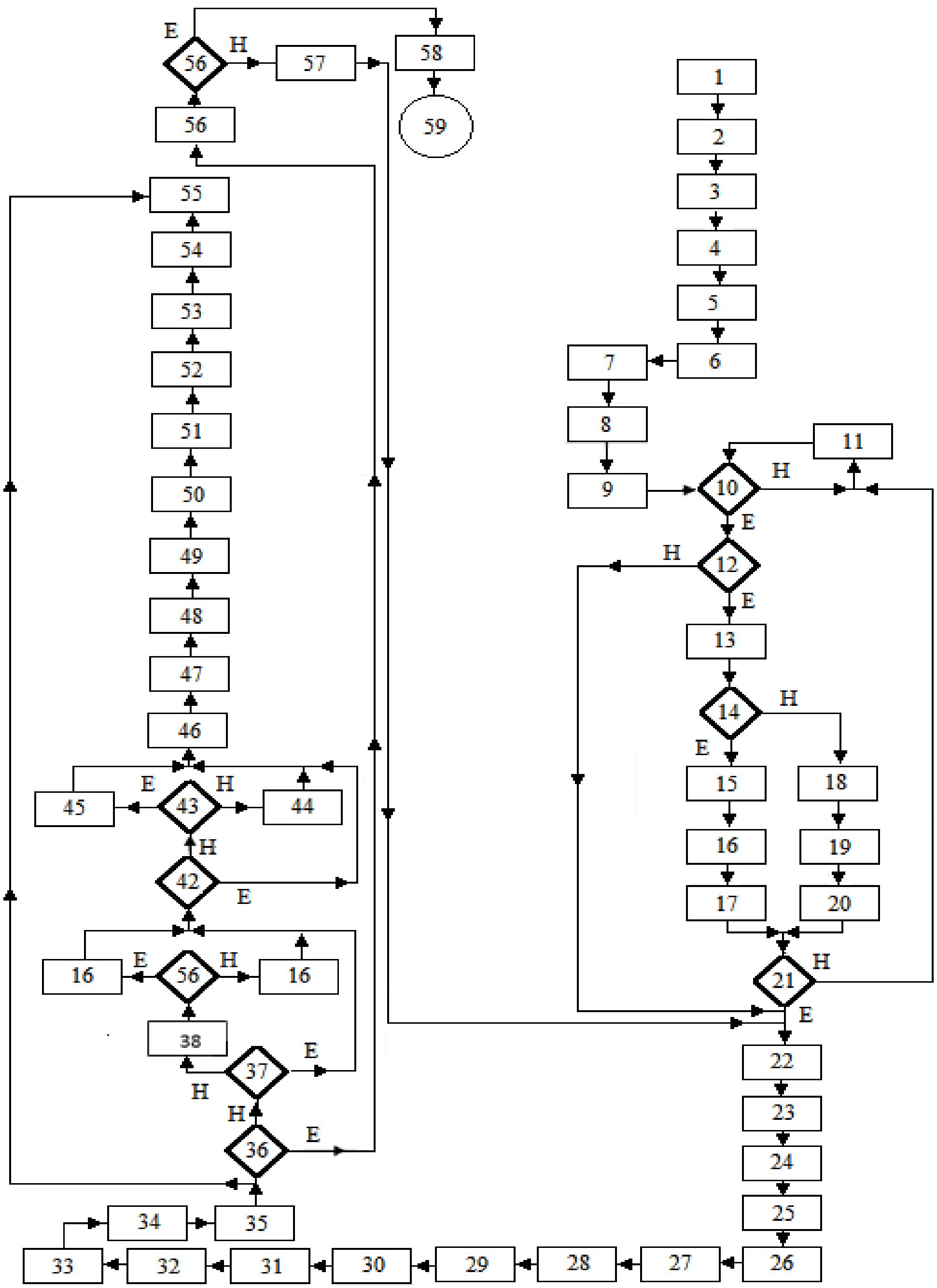

Figure 4. Flowchart of the design of air core tapped reactor (continued)

1- Input frequency (Hz), nominal current value (A) of the tapped reactor, maximum inductance and tolerance of maximum inductance (mH) of the tapped reactor.

2- Input number of taps, inductance values from lower to higher for each taps
3- Input material of wire of layers (Al or Cu) and dencity (A/mm^2) of currents of the layers.
4- Input beginning inner diameter (cm) and the last inner diameter (cm) of the tapped reactor.
5- Input the package number and number of parallel connected layers in a package of the tapped reactor.
6- Use the lower inductance value as inductance of the tapped reactor.
7- Input thickness (mm) of the paper between two layers and thickness (cm) of a dog bone.
8- Input diameters (mm) of layers from the outer layer to the inner layer wire with izolation or not respectively.
9- Input allowable number of iterations (k) and the initial value of the iteration is 1.
10- Do all layers withstand the current (A) of the tapped reactor?
11- Input the new wire diameters (mm) of the layers.
12- Is it important of short circuit current for the tapped reactor?
13- Input the short circuit current (A) of the tapped reactor.
14- Was copper used for layer wire?
15- Input beginning and the last temperature of the layer (C^{o}).
16- Input short circuit time (sn) of the reactor.
17- Calculate current density (A/mm^2) of the short circuit current of the reactor using Eq.(23).
18- Input beginning and the last temperature of the layer (C^{o}).
19- Input short circuit time (sn) of the tapped reactor.
20- Calculate current density (A/mm^2) of the short circuit current of the tapped reactor using Eq.(22).
21- Using Eq.(24), do all layers withstand the short circuit current of the tapped reactor?
22- Input acceptable lower and higher limits of the tapped reactor for the current tap value.
23- Accept inductance value (mH) of the outer layer of the reactor 1.25 times as more inductance as the inductance of the tapped reactor for the current tap value.
24- Using Eqs. (25-27), calculate number of spirs of the outer layer of the tapped reactor for the current tap value.
25- Guess number of spirs of other layers as the number of spir of a layer is %20 less than the one of its outer layer.
26- Calculate voltage (V) of the tapped reactor using both empedance and current of the tapped reactor for the current tap value.
27- Calculate estimated height (cm) of the tapped reactor for the current tap value.
28- Calculate diameters (cm) of the all layers for the current tap value.
29- Calculate of inductance (mH) using Eq.(6), average diameter (cm), number of spirs and height of layer (cm) for all layers of the tapped reactor for the current tap value.
30- Calculate mutual inductances (mH) using Eqs. (15-21) among all layers using average diameter (cm), number of spirs and height of layer (cm) for all layers of the tapped reactor for the current tap value.
31- Calculate of dc resistances (ohm) of all layers for both 25 C^{o} and 75 C^{o} for the current tap value.
32- Calculate impedance matrix (ohm) of the reactor using Eq.(1b) for the current tap value.
33- Using Eq.(1b), calculate the currents (A) of all layers using applied voltage (V) and impedance matrix (ohm) of the tapped reactor for the current tap value.
34- Using Eq.(2), calculate current (A) of the tapped reactor summing all currents of the parallel connected layers for the current tap value.
35- Calculate inductance (mH) of the tapped reactor of the reactor using current (A), voltage (V) and frequency (Hz) for the current tap value.
36- Is current (A) value of the tapped reactor between allowable limit value? and is current (A) of each layer between allowable upper and lower limit values? and is the number of iteration less than allowable upper limit value? (All questions are for the current tap value).
37- Are currents (A) of all layers between acceptable their own limit values?
38- Choose layers overflowing their own nominal current values. Don’t change number of spirs of the layers which current values don’t overflow their nominal limit values of currents.

39- Is the current (A) of the layer less than its own lower limit value?

Figure 4. Flowchart of the design of air core tapped reactor (continued)

40- Add spirs using coefficient (depending on iteration number) to the layer which current value ise more than acceptable upper limit value of the current.
41- Reduce spirs using coefficient (depending on iteration number) to the layer which current value ise less than acceptable lower limit value of the current.
42- Is the current value (A) of the reactor between upper and lower limit value?
43- Is the current value (A) of the reactor less than upper limit value?
44- Reduce upper limit value of current (A) of the tapped reactor, using coefficient depending on iteration number.
45- Raise lower limit value of current (A) of the tapped reactor, using coefficient depending on iteration number.
46- Calculate height (cm) of the tapped reactor using recalculated number of spirs of the layers.
47- Calculate self inductances (mH) of all layers using recalculated spir numbers of all layers, diameters (cm) of all layers, height (cm) of all layers Eq.(6).
48- Calculate mutual inductances (mH) among all layers using recalculated number of spirs of all layers using Eq.(15), diameters (cm) of all layers, heights (cm) of all layers, self inductances (mH) of all layers using Eq.(6).
49- Calculate of dc resistances (ohm) of all layers.
50- Form the impedance matrix of the tapped reactor using Eq.(1b).
51- Calculate currents (A) of all layers using applied voltage (V) and impedance matrix of the tapped reactor using Eq.(1b) for the current tap value.
52- Calculate current (A) of the tapped reactor adding currents of all layers for the current tap value.
53- Calculate self inductance (mH) of the tapped reactor using current (A), voltage (V) and frequency (hz) of the tapped reactor.
54- Increase iteration number
55- Write the currents, number of spirs and height of layers for the current tap value.
56- Are all taps of the tapped reactor finished?
57- Increase the current tap value as 1.
58- Calculate current value (A) of the tapped reactor, the angle (degree) of current of the reactor, dc resistance (ohm) of the reactor both 25 C^o and 75 C^o, reactive power of the tapped reactor (kVAr), quality factor of the tapped reactor, number of spirs of all layers, total length of the wire (m) of the all layers, total power loss (kW) of the tapped reactor, total weight (kg) of the reactor, height (cm) of the reactor, total weight (kg) of the wire of the tapped reactor, inductance (mH) of the reactor.
59- Stop

Figure 4. Flowchart of the design of air core tapped reactor

6. Formulation of number of spirs of the outer layer

It can be calculated number of spirs (N) of the outer layer using the formulation for tap k;

$$N = (L_1 10^6) - A_1 + A_2 \quad (25)$$

In the Eq.(25), L_1 is self inductance of the outer layer (henry). A_1 and A_2 have been given in Eqs.(26-27);

$$A_1 = 0.002\pi D_1 N^2 (\log(1 + \frac{0.5\pi D_1}{dN})) \tag{26}$$

$$A_2 = \frac{1}{2.3 + (3.437 dN / D_1) + 1.7636 (dN / D_1)^2 - 0.47((0.755 + (dN / D_1))^{1.44}} \tag{27}$$

In Eqs.(26-27), d;diameter of the wire with izolation (cm), D_1;internal diameter of the reactor (cm) for tap k. Flowchart of the design of the air core tapped reactor have been given in fig. 6.

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