Internal Stability Loss of a Periodical Curved Carbon Nanotube in a Viscoelastic Matrix

*Reşat Köşker and İsmail Gülten
Yildiz Technical University, Davutpasa Campus, Fac. of Che. and Me., Dep. of Mathematical Engineering, Turkey

Abstract

Within the framework of the piecewise homogeneous body model with the use of the Three-Dimensional Geometrically Nonlinear Exact Equations of the Theory of Viscoelasticity the approach for the investigation of the internal stability loss (microbuckling) in the structure of the viscoelastic unidirected fibrous nanocomposites under compression along the fibers is developed in this work. This development concerns mainly the case where the viscoelastic matrix contains a periodical curved carbon nanotube (CNT) and It is assumed that the CNT have an initial infinitesimal imperfection. The form of this imperfection is taken as periodical curving of the CNT. The growing of the initial imperfection is investigated under fixed compressed external forces. Using the developed approach the numerical results related to the critical time are presented. In this case, the influence of the rheological parameters of the matrix material to the values of the critical time is also investigated.

Key words: Fracture in compression, Nanocomposite, Nanotube, Critical time, Internal stability loss

1. Introduction

It is known that one of the major mechanisms of the fracture of the unidirectional composites under uniaxial compression along the reinforcing elements is the stability loss in the material structure. According to this mechanism the theoretical investigations of the fracture of the unidirectional composites under uniaxial compression along the reinforcing elements are reduced to the investigations of the stability loss in the material structure, and the value of the external critical force is accepted as the value of failure force. At present, numerous theoretical investigations have been carried out in this field. The review of these investigations is given in [1-3]. Currently, in investigations of the stability and fracture of composite materials under compression along the reinforcing elements in the framework of a piecewise-homogeneous body model, essentially two approaches are used: application of certain hypotheses related to deformation of single components and to the character of interacting between them; and application of the Three-Dimensional Linearized Theory of Stability (TDLTS). It is evident that the results on the considered problems obtained within the framework of the TDLTS are more accurate than those obtained within the framework of the approximate theories. However, the investigations carried out in the framework of the TDLTS and listed in [1- 3] relate to the time-independent materials. In the paper [4] in the framework of the TDLTS the approach for the investigation of the stability loss in the time-dependent layered composite material is proposed. In the paper [5], the approach [4] is developed for the unidirected fibrous composite material. However in [5] it is assumed that the filler concentration in the composite is very small and

*Corresponding author: Address: Yildiz Technical University, Davutpasa Campus, Fac. of Che. and Me., Dep. of Mathematical Engineering, 34210, Istanbul TURKEY. E-mail address: kosker@yildiz.edu.tr, Phone: +902123834607 Fax: +902123834607

interacting between the fibers is ignored. Consequently, the material is modeled as an infinite viscoelastic body containing a single fiber. In [6], the approach proposed in [5] is developed for the case where taken the interactions between the fibers into account, and the corresponding analysis were made on the infinite viscoelastic media containing two neighboring fibers. However in all investigations detailed in the monograph [7], it was assumed that the reinforcing elements of composite materials are made of traditional materials. In the paper [8] the attempt is made for development of the internal stability loss problems in the structure of the unidirectional fibrous composites for the case where the reinforcing element in the composite is the double-walled carbon nanotube (DWCNT). In this study, the case is considered where a single periodical curved carbon nanotube (CNT) with an infinite length is contained by an infinite body with low concentration of fibers and the stability loss problem in that is investigated. Taking the low concentration of CNT into account the interaction between them is neglected. In this case we assume that the CNT has an initial insignificant periodical imperfection and investigate the grow of this imperfection with flow a time. The case where this imperfection starts to increase and becomes indefinitely is taken as a stability loss criterion and from this criterion the critical time is determined. The investigations are carried out within the framework of the piecewise homogeneous body model with the use of the three-dimensional geometrical nonlinear exact equations of the theory of viscoelasticity.

2. Formulation of the Problem

We consider an infinite body containing a single CNT with insignificant initial imperfection (Fig.1). The values related to the CNT denote by upper index (2), the values related to a matrix by upper index (1). With the middle line of the CNT we associate Lagrangian rectilinear $Ox_1x_2x_3$ and cylindrical $Or\theta z$ system of coordinates and the material of the CNT and matrix we take as homogeneous, isotropic and non-aging linear viscoelastic. Moreover we assume that this body is compressed at $|x_3| \to \infty$ by uniformly distributed normal forces with intensity p acting along the Ox_3 axis and the cross section of the CNT normal to the middle line of the CNT is circle, with constant radius R+H and with constant thickness H along the entire length of the CNT. Thus within the CNT and matrix in the geometrical non-linear statement we write the governing field equations:

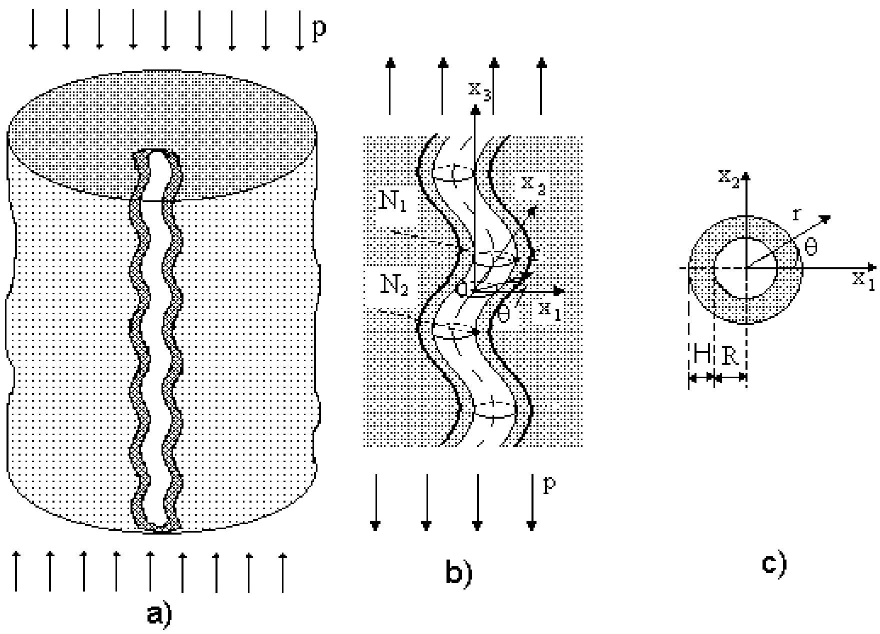
$$\nabla_i\left[\sigma^{(k)in}\left(g_n^j+\nabla_n u^{(k)j}\right)\right]=0,\ 2\varepsilon_{jm}^{(k)}=\nabla_j u_m^{(k)}+\nabla_m u_j^{(k)}+\nabla_j u^{(k)n}\nabla_m u^{(k)n},$$

$$\sigma_{(in)}^{(k)}=\left(\lambda_0^{(k)}e^{(k)}(t)+\int_0^t\lambda^{(k)}(t-\tau)e^{(k)}(\tau)d\tau\right)\delta_i^n+2\left(\mu_0^{(k)}\varepsilon_{(in)}^{(k)}(t)+\int_0^t\mu^{(k)}(t-\tau)\varepsilon_{(in)}^{(k)}d\tau\right),$$

$$e^{(k)}=\varepsilon_{(11)}^{(k)}+\varepsilon_{(22)}^{(k)}+\varepsilon_{(33)}^{(k)}. \tag{1}$$

It is assumed that on the inter-medium surface S the completely cohesion conditions are satisfied:

$$\sigma^{(1)in}\left(g_n^j+\nabla_n u^{(1)j}\right)\Big|_S n_j=\sigma^{(2)in}\left(g_n^j+\nabla_n u^{(2)j}\right)\Big|_S n_j,\ u^{(1)j}\Big|_S=u^{(2)j}\Big|_S, \tag{2}$$



In the considered case it is also assumed that the conditions $\left|\sigma^{(2)}_{(ij)}\right| < \infty$, $\left|u^{(2)}_{(i)}\right| < \infty$, $\sigma^{(1)}_{zz} \xrightarrow[r\to\infty]{} p$, $\sigma^{(1)}_{(ij)} \xrightarrow[r\to\infty]{} 0$ ($(ij) \neq zz$) are satisfied. Moreover, in the inner surface of CNT (denoted by S_0

Figure 1. The geometry of the material structure and chosen coordinates

and n_j^0 are the component of the normal vector of S_0) the following conditions can be written:

$$\sigma^{(2)in}\left(g_n^j + \nabla_n u^{(2)j}\right)\Big|_{S_0} n_j^0 = 0\,, \tag{3}$$

In (1-3) the conventional tensor notation is used and subindexes in parentheses show the physical components of the corresponding tensors. Writing the expression of the Lamé's coefficients in the cylindrical coordinate system and doing some performing we can obtain the expression of the equations (1-3) in the cylindrical system of coordinates. The initial imperfection form of the CNT we give via the following equation of the middle line of that

$$x_2 = F(x_3) = \varepsilon\delta(x_3)\,,\; x_1 = 0\,, \tag{4}$$

where ε is a small parameter. The geometrical significance of this parameter will be indicated with a specifically prescribed form of the function (4). Thus, the investigation of the development of the CNT initial imperfection (4) is reduced to the solution of the equation (1) in within the contact conditions (2-3).

3. Method of Solution

Using (4) and the condition on the cross section of the CNT, we derive the following equation of the inter-surface S in the cylindrical system of coordinates $Or\theta z$.

$$r = \left(1+\varepsilon^2\left(\delta'(t_3)\right)^2\sin^2\theta\right)^{-1}\left\{\left(\varepsilon\delta(t_3)+\varepsilon^3\delta(t_3)\left(\delta'(t_3)\right)^2\right)\sin\theta + \right.$$

$$\left[R^2-\varepsilon^2\left(\delta(t_3)\right)^2-\varepsilon^4\left(\delta'(t_3)\right)^2\left(\delta(t_3)\right)^2\left(1+\varepsilon^2\left(\delta'(t_3)\right)^2\right)\sin^2\theta\right]^{1/2}\Bigg\},$$

$$z=t_3-\varepsilon\delta'(t_3)r(t_3)\sin\theta+\varepsilon^2\delta(t_3)\delta'(t_3)\,, \tag{5}$$

where t_3 is a parameter ($t_3\in(-\infty,+\infty)$). After certain transformations, we obtain the expressions from (5) for the components n_r, n_θ, n_3 of the unite normal vector to the surface S. Taking into account that the initial imperfection of the CNT has an infinite small size we seek quantities characterizing the stress-strain state of the matrix and CNT in the form of series in positive powers of the small parameter ε

$$\sigma_{rr}^{(k)}=\sum_{q=0}^{\infty}\varepsilon^q\sigma_{rr}^{(k),q}\,,\ldots,\ \varepsilon_{rr}^{(k)}=\sum_{q=0}^{\infty}\varepsilon^q\varepsilon_{rr}^{(k),q}\,,\ldots,\ u_r^{(k)}=\sum_{q=0}^{\infty}\varepsilon^q u_r^{(k),q}\,,. \tag{6}$$

We represent the expressions (5) and the expressions of n_r, n_θ, n_3 in the following series form.

$$r=R+\sum_{q=1}^{\infty}\varepsilon^q a_{rq}(R,\theta,t_3)\,,\quad z=t_3+\sum_{q=1}^{\infty}\varepsilon^q a_{zq}(R,\theta,t_3)\,,$$

$$n_r=1+\sum_{q=1}^{\infty}\varepsilon^q b_{rq}(R,\theta,t_3)\,,\qquad n_\theta=\sum_{q=1}^{\infty}\varepsilon^q b_{\theta q}(R,\theta,t_3)\,,\quad n_z=\sum_{q=1}^{\infty}\varepsilon^q b_{zq}(R,\theta,t_3)\,. \tag{7}$$

The expressions for the coefficients of the ε^q in (7) can be determined by employing routine operations; we omit details. Thus, substituting the expression (6) in (1) and grouping by identical powers we obtain the completely system equations for each approximation. In this case for the zeroth approximation the equation (1) hold and the equations derived for the first and subsequent approximation contain the values of the previous approximations. We assume that the materials of the both matrix and CNT is comparatively rigid and in this base the non-linear terms can be neglected in the equations obtained for the zeroth approximation and for the first and subsequent approximations the term $(g_n^j+\nabla_n u^{(k)j,0})$ can be replaced by δ_n^j. By direct verification we prove that the equations obtained for the first approximation coincide with the equations of the Three-Dimensional Linearized Theory of Stability (TDLTS) [9]. It should be noted that the homogeneous parts of the equations obtained for the second and for the subsequent approximations also coincide with the equations of the TDLTS. Consider the contact conditions for each approximation which are derived from the (2) and (3). For this purpose we substitute the expression (6), (7) in (2)-(3) and expand the components of each approximation of (6) in Taylor's series in the vicinity $(R+H,\theta,z)$. Then, grouping by identical powers of the parameter ε and taking into account the foregoing assumptions we derive the contact conditions for each approximation. We record them for the zeroth and first approximations.

The zeroth approximation.

$$\left[\sigma_{rr}\right]^{2,0}=\left[\sigma_{r\theta}\right]^{2,0}=\left[\sigma_{rz}\right]^{2,0}=0,\ \left[\sigma_{rr}\right]_{1,0}^{2,0}=\left[\sigma_{r\theta}\right]_{1,0}^{2,0}=\left[\sigma_{rz}\right]_{1,0}^{2,0}=0,\left[u_r\right]_{1,0}^{2,0}=\left[u_\theta\right]_{1,0}^{2,0}=\left[u_z\right]_{1,0}^{2,0}=0 \tag{8}$$

The first approximation.

$$\left[\sigma_{rr}\right]^{2,1}+f_1\left[\frac{\partial\sigma_{rr}}{\partial r}\right]^{2,0}+\phi_1\left[\frac{\partial\sigma_{rr}}{\partial z}\right]^{2,0}+\gamma_r\left[\sigma_{rr}\right]^{2,0}+\gamma_\theta\left[\sigma_{r\theta}\right]^{2,0}+\gamma_z\left[\sigma_{rz}\right]^{2,0}=0$$

$$\left[\sigma_{rr}\right]_{1,1}^{2,1}+f_1\left[\frac{\partial\sigma_{rr}}{\partial r}\right]_{1,0}^{2,0}+\phi_1\left[\frac{\partial\sigma_{rr}}{\partial z}\right]_{1,0}^{2,0}+\gamma_r\left[\sigma_{rr}\right]_{1,0}^{2,0}+\gamma_\theta\left[\sigma_{r\theta}\right]_{1,0}^{2,0}+\gamma_z\left[\sigma_{rz}\right]_{1,0}^{2,0}=0,$$

$$\left[u_r\right]_{1,1}^{2,1}+f_1\left[\frac{\partial u_r}{\partial r}\right]_{1,0}^{2,0}+\phi_1\left[\frac{\partial u_r}{\partial z}\right]_{1,0}^{2,0}=0. \tag{9}$$

where the following notation is used.

$$\left[\mathrm{X}\right]^{2,q}=\mathrm{X}^{(2),q}\left(R,\theta,z\right),\ \left[\mathrm{X}\right]_{1,q}^{2,q}=\mathrm{X}^{(1),q}\left(R+H,\theta,z\right)-\mathrm{X}^{(2),q}\left(R+H,\theta,z\right),\quad q=0,1,\ \ f_1=\delta(t_3)\sin\theta,$$

$$\phi_1=-R\frac{d\delta(t_3)}{dt_3}\sin\theta,\ \gamma_r=\left(\frac{\delta(t_3)}{R}-\frac{d^2\delta(t_3)}{dt_3^2}R\right)\sin\theta,\ \gamma_\theta=-\frac{\delta(t_3)}{R}\cos\theta,\ \ \gamma_z=-\frac{d\delta(t_3)}{dt_3}\sin\theta. \tag{10}$$

Moreover, in (9) we have given the contact relations for radial force $\left(\sigma_{rr}n_r+\sigma_{r\theta}n_\theta+\sigma_{rz}n_z\right)$ and radial displacement u_r. The rest of the contact relations for the first approximation are obtained from (9) by means of cyclic permutation of the indices r,θ and z only in the components stress tensor (the first index is permuted) and displacement vector. Consider the determination of each approximation.

The zeroth approximation. In the pure elastic case this approximation has the exact analytical solution [9]. Replacing the elastic constants in this solution with corresponding operators we obtain the solution to the quasistatic problem related to the zeroth approximation. We obtain the following relations from the mentioned solution under the assumption that $\nu^{(1)}=\nu^{(2)}$ [9].

$$\varepsilon_{zz}^{(1),0}=\varepsilon_{zz}^{(2),0}=\frac{p}{E^{(1)}},\ \sigma_{zz}^{(1),0}=p,\ u_z^{(1),0}=u_z^{(2),0}=\frac{p}{E^{(1)}}z,\ u_r^{(1),0}=-\nu^{(1)}\varepsilon_{zz}^{(1),0},$$

$$u_r^{(2),0}=-\nu^{(2)}\varepsilon_{zz}^{(2),0},\ u_\theta^{(1),0}=u_\theta^{(2),0}=0,\ \sigma_{rr}^{(1)}=\sigma_{rr}^{(2)}=\sigma_{\theta\theta}^{(1)}=\sigma_{\theta\theta}^{(2)}=0,\ \sigma_{zz}^{(2)}=p\frac{E^{(2)}}{E^{(1)}}. \tag{11}$$

The first approximation. The equation (4) is selected as follows

$$x_2=L\sin\frac{2\pi}{\ell}x_3=\varepsilon\ell\sin\alpha x_3,\ \ x_1=0;\qquad \delta(x_3)=\varepsilon\sin\alpha x_3,\ L<<\ell,\ \ \varepsilon=\frac{L}{\ell} \tag{12}$$

Assume that the material of the CNT is pure elastic and the following operators describe the behavior of the matrix material

$$E^{(1)}=E_0^{(1)}\left[1-\omega_0R_{\alpha'}^*(-\omega_0-\omega_\infty)\right],\ \ \nu^{(1)}=\nu_0^{(1)}\left[1+\frac{1-2\nu_0^{(1)}}{2\nu_0^{(1)}}\omega_0R_{\alpha'}^*(-\omega_0-\omega_\infty)\right], \tag{13}$$

where $E_0^{(1)}$, $\nu_0^{(1)}$ are the momentary values of the Young modules and Poisson's ratio, respectively; α', ω_0 and ω_∞ are rheological parameters; $R_{\alpha'}^*$ is the fractional-exponential operator of Rabotnov [10].

According to (11) the equations obtained for the first approximation have the following form

$$\frac{\partial\sigma_{rr}^{(k),1}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}^{(k),1}}{\partial\theta}+\frac{\partial\sigma_{rz}^{(k),1}}{\partial z}+\frac{1}{r}\left(\sigma_{rr}^{(k),1}-\sigma_{\theta\theta}^{(k),1}\right)+\sigma_{zz}^{(k),0}\frac{\partial^2u_r^{(k),1}}{\partial z^2}=0,$$

$$\frac{\partial\sigma_{r\theta}^{(k),1}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}^{(k),1}}{\partial\theta}+\frac{\partial\sigma_{\theta z}^{(k),1}}{\partial z}+\frac{2}{r}\sigma_{r\theta}^{(k),1}+\sigma_{zz}^{(k),0}\frac{\partial^2u_\theta^{(k),1}}{\partial z^2}=0,$$

$$\frac{\partial\sigma_{rz}^{(k),1}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}^{(k),1}}{\partial\theta}+\frac{\partial\sigma_{zz}^{(k),1}}{\partial z}+\frac{1}{r}\sigma_{rz}^{(k),1}+\sigma_{zz}^{(k),0}\frac{\partial^2 u_z^{(k),1}}{\partial z^2}=0\,, \tag{14}$$

Due to pure elasticity of the CNT material the relations (1) for that are replaced by the following ones.

$$\sigma_{(in)}^{(2),1}=\lambda^{(2)}e^{(2),1}\delta_i^n+2\mu^{(2)}\varepsilon_{(in)}^{(2),1}. \tag{15}$$

It follows from (11), (14) and (15) that *t* (time) is a parameter in the equations related to the CNT. However *t* is an independent variable in the equations related to the matrix. According to this situation we employ the Laplace transform

$$\bar{\phi}(s)=\int_0^\infty \phi(t)\,e^{-st}dt \tag{16}$$

with parameter $s>0$, to all equations and relations (excepting the contact relations (9)) related to the matrix material. After this procedure the equations (14) and others are valid for the Laplace transforms of the south values, however the constitutive relations obtained for first approximation are transformed to the following ones.

$$\bar{\sigma}_{(in)}^{(1),1}=\bar{\lambda}^{(1)}\bar{e}^{(1),1}\delta_i^n+2\bar{\mu}^{(1)}\bar{\varepsilon}_{(in)}^{(1),1},\quad \bar{\lambda}^{(1)}=\frac{\bar{E}^{(1)}\bar{\nu}^{(1)}}{(1+\bar{\nu}^{(1)})(1-2\bar{\nu}^{(1)})},\quad \bar{\mu}^{(1)}=\frac{\bar{E}^{(1)}}{2(1+\bar{\nu}^{(1)})}. \tag{17}$$

For the solution of the system equations coupled from (14), (15), (17) for the CNT and the system equations coupled from the Laplace transforms of (14) and from the (17) we employ the following representations [9].

$$u_r=\frac{1}{r}\frac{\partial}{\partial\theta}\Psi-\frac{\partial^2}{\partial r\partial z}\Upsilon\,,\ u_\theta=-\frac{\partial}{\partial r}\Psi-\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\Upsilon\,,\ u_3=(\lambda+\mu)^{-1}\left((\lambda+2\mu)\Delta_1+\mu\frac{\partial^2}{\partial z^2}\right)\Upsilon\,. \tag{18}$$

The functions Ψ and Υ are determined from the equations

$$\left(\Delta_1+\xi_1^2\frac{\partial^2}{\partial z^2}\right)\Psi=0\,,\ \left(\Delta_1+\xi_2^2\frac{\partial^2}{\partial z^2}\right)\left(\Delta_1+\xi_3^2\frac{\partial^2}{\partial z^2}\right)\Upsilon=0\,, \tag{19}$$

where

$$\xi_1=\sqrt{\frac{\mu+\sigma_{zz}^0}{\mu}}\,,\ \xi_2=\sqrt{\frac{\mu+\sigma_{zz}^0}{\mu}}\,,\ \xi_3=\sqrt{\frac{\lambda+2\mu+\sigma_{zz}^0}{\lambda+2\mu}}\,,\quad \Delta_1=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\,. \tag{20}$$

The equations (18)-(20) have been written without upper indexes. We use these relations for the matrix (CNT) and in this case we replace the values $u_{(i)}$, λ, μ and σ_{zz}^0 by $\bar{u}_{(i)}^{(1)}$, $\bar{\lambda}^{(1)}$, $\bar{\mu}^{(1)}$ and $\sigma_{zz}^{(1),0}$ ($u_{(i)}^{(2)}$, $\lambda^{(2)}$, $\mu^{(2)}$ and $\sigma_{zz}^{(2),0}$) respectively.

Taking (9), (10), (11) and (12) into account the contact conditions for the first approximation can be represented as follows.

$$\left[\sigma_{rr}\right]^{2,1}=0\,,\ \left[\sigma_{r\theta}\right]^{2,1}=0\,,\ \left[\sigma_{rz}\right]^{2,1}=2\pi\sigma_{zz}^{(2),0}\cos\alpha t_3\sin\theta\,,\ \left[\sigma_{rr}\right]_{1,1}^{2,1}=0\,,\ \left[\sigma_{r\theta}\right]_{1,1}^{2,1}=0\,,$$

$$\left[\sigma_{rz}\right]_{1,1}^{2,1}=2\pi\left(\sigma_{zz}^{(1),0}-\sigma_{zz}^{(2),0}\right)\cos\alpha t_3\sin\theta\,,\ \left[u_r\right]_{1,1}^{2,1}=0\,,\ \left[u_\theta\right]_{1,1}^{2,1}=0\,,\ \left[u_z\right]_{1,1}^{2,1}=0\,. \tag{21}$$

Duo to (21) we define the solution to the equations (21) as follows.

$$\bar{\Psi}^{(1)}=\bar{A}_1^{(1)}(s)I_1\left(\xi_1^{(1)}(s)\alpha r\right)\sin\alpha z\cos\theta\,,$$

$$\bar{\Upsilon}^{(1)} = \left[\bar{A}_2^{(1)}(s) I_1\left(\xi_2^{(1)}(s)\alpha r\right) + \bar{A}_3^{(1)}(s) I_1\left(\xi_3^{(1)}(s)\alpha r\right)\right] \cos\alpha z \sin\theta , \tag{22}$$

$$\Psi^{(2)} = \left[A_1^{(2)}(t) K_1\left(\xi_1^{(2)}(t)\alpha r\right) + B_1^{(2)}(t) \mathrm{I}_1\left(\xi_1^{(2)}(t)\alpha r\right)\right] \sin\alpha z \cos\theta ,$$

$$\Upsilon^{(2)} = \begin{bmatrix} A_2^{(2)}(t) K_1\left(\xi_2^{(2)}(t)\alpha r\right) + A_3^{(2)}(t) K_1\left(\xi_3^{(2)}(t)\alpha r\right) + \\ B_2^{(2)}(t) I_1\left(\xi_2^{(2)}(t)\alpha r\right) + B_3^{(2)}(t) \mathrm{I}_1\left(\xi_3^{(2)}(t)\alpha r\right) \end{bmatrix} \cos\alpha z \sin\theta , \tag{23}$$

where $I_1(x)$ is a Bessel function of a purely imaginary argument and $K_1(x)$ is a Macdonald function. Using (15), (17), (18), (22) and (23) we determine the values related to the CNT via the unknowns $A_1^{(2)}(t)$, $A_2^{(2)}(t)$ and $A_3^{(2)}(t)$, and the Laplace transform of the values related to the matrix via the unknowns $\bar{A}_1^{(1)}(s)$, $\bar{A}_2^{(1)}(s)$ and $\bar{A}_3^{(1)}(s)$. Now we consider the determination of the inverse of Laplace transform. For this purpose we use Schapery method [11]. So, we determine the unknowns $A_1^{(2)}(t)$, $A_2^{(2)}(t)$, $A_3^{(2)}(t)$ and $\bar{A}_1^{(1)}(s)$, $\bar{A}_2^{(1)}(s)$, $\bar{A}_3^{(1)}(s)$ at $s = 1/(2t)$ for any selected t and from the criterion $\max\limits_{z\in[0,\ell];\theta\in[0,\pi/2]} u_r^{(2),1} \to \infty$ we determine the critical time. In this criterion we use only the first approximation because the second and subsequent approximations do not change the values of the critical time.

3. Numerical Results and Discussion

We introduce the dimensionless rheological parameter $\omega(=\omega_\infty/\omega_0)$ and the dimensionless time $t'\left(=\omega_0^{1/(1+\alpha')}t\right)$ and assume that $\nu_0^{(1)} = \nu^{(2)} = 0.3$. It is well known that under investigation of stability loss problems for viscoelastic materials the external compressive force p must satisfy the following inequalities

$$\in_{cr.\infty}\left(= p_{cr.\infty}/E_0^{(1)}\right) \le \in\left(= p/E_0^{(1)}\right) \le \in_{cr.0}\left(= p_{cr.0}/E_0^{(1)}\right). \tag{24}$$

The values of $\in_{cr.\infty}$ (for $t' = \infty$) and of $\in_{cr.0}$ (for $t' = 0$) with various $E = E^{(2)}/E_0^{(1)}$ (500 and 700), $\gamma_1 = 2\pi(R+H)/\ell = \alpha(R+H)$, $\gamma_2 = H/(R+H)$ and ω are given in Tables 1-4. Note that under calculation of $\varepsilon_{cr.\infty}$ and $\varepsilon_{cr.0}$ the purely elastic problems are solved and in these case the operators $E^{(1)}$, $\nu^{(1)}$ are replaced with the constants $E_\infty^{(1)}\left(= E^{(1)}\big|_{t'=\infty}\right)$, $\nu_\infty^{(1)}\left(= \nu^{(1)}\big|_{t'=\infty}\right)$, and $E_0^{(1)}\left(= E^{(1)}\big|_{t'=0}\right)$, $\nu_0^{(1)}\left(= \nu^{(1)}\big|_{t'=0}\right)$ respectively. It follows from these results that there is non-monotonic character between the parameters γ_1 and $\in_{cr,0}$, $\in_{cr,\infty}$, but the values of $\in_{cr,0}$ and $\in_{cr,\infty}$ decrease with γ_2. However the values of $\in_{cr,\infty}$ increase with ω. The values of $\in_{cr,0}$ and $\in_{cr,\infty}$ decrease with *E*.

Table 1. The values of $\in_{cr.\infty}$ and of $\in_{cr.0}$ for $E = 500$, $\gamma_2 = 0.3$, $\alpha' = -0.5$ with various γ_1

γ_1	$\in_{cr,0}$	$\in_{cr,\infty}$

		$\omega = 0.5$	$\omega = 1.0$	$\omega = 2.0$	$\omega = 3.0$
0.1	-0.1597	-0.0591	-0.0850	-0.1102	-0.1227
0.2	-0.0691	-0.0323	-0.0414	-0.0505	-0.0551
0.3	-0.0589	-0.0384	-0.0434	-0.0484	-0.0510
0.4	-0.0655	-0.0519	-0.0552	-0.0585	-0.0602
0.5	-0.0775	-0.0676	-0.0699	-0.0724	-0.0736
0.6	-0.0910	-0.0832	-0.0851	-0.0870	-0.0879
0.7	-0.1041	-0.0979	-0.0994	-0.1009	-0.1017
0.8	-0.1163	-0.1110	-0.1123	-0.1135	-0.1142
0.9	-0.1270	-0.1225	-0.1236	-0.1247	-0.1252
1.0	-0.1362	-0.1324	-0.1333	-0.1342	-0.1347

Table 2. The values of $\in_{cr.\infty}$ and of $\in_{cr.0}$ for $E = 500$, $\gamma_1 = 0.3$, $\alpha' = -0.5$ with various γ_2

γ_2	$\in_{cr,0}$	$\in_{cr,\infty}$			
		$\omega = 0.5$	$\omega = 1.0$	$\omega = 2.0$	$\omega = 3.0$
0.010	-0.3845	-0.2304	-0.2915	-0.3337	-0.3490
0.015	-0.3822	-0.1785	-0.2336	-0.2798	-0.2998
0.020	-0.3703	-0.1478	-0.1949	-0.2371	-0.2566
0.025	-0.3493	-0.1278	-0.1683	-0.2059	-0.2238
0.030	-0.3254	-0.1138	-0.1492	-0.1827	-0.1989
0.035	-0.3024	-0.1034	-0.1348	-0.1649	-0.1796
0.040	-0.2814	-0.0954	-0.1236	-0.1509	-0.1642
0.045	-0.2629	-0.0891	-0.1146	-0.1395	-0.1518
0.050	-0.2466	-0.0839	-0.1073	-0.1302	-0.1415

Table 3. The values of $\in_{cr.\infty}$ and of $\in_{cr.0}$ for $E = 700$, $\gamma_2 = 0.3$, $\alpha' = -0.5$ with various γ_1

γ_1	$\in_{cr,0}$	$\in_{cr,\infty}$			
		$\omega = 0.5$	$\omega = 1.0$	$\omega = 2.0$	$\omega = 3.0$
0.1	-0.1188	-0.0437	-0.0627	-0.0815	-0.0908
0.2	-0.0538	-0.0271	-0.0336	-0.0402	-0.0436
0.3	-0.0503	-0.0356	-0.0391	-0.0428	-0.0446
0.4	-0.0599	-0.0501	-0.0524	-0.0548	-0.0560
0.5	-0.0734	-0.0662	-0.0679	-0.0697	-0.0706
0.6	-0.0878	-0.0822	-0.0835	-0.0849	-0.0856
0.7	-0.1016	-0.0971	-0.0981	-0.0992	-0.0998
0.8	-0.1141	-0.1104	-0.1112	-0.1122	-0.1126
0.9	-0.1252	-0.1220	-0.1227	-0.1235	-0.1239
1.0	-0.1347	-0.1319	-0.1326	-0.1332	-0.1336

Table 4. The values of $\in_{cr.\infty}$ and of $\in_{cr.0}$ for $E = 700$, $\gamma_1 = 0.3$, $\alpha' = -0.5$ with various γ_2

γ_2	$\in_{cr,0}$	$\in_{cr,\infty}$			
		$\omega = 0.5$	$\omega = 1.0$	$\omega = 2.0$	$\omega = 3.0$
0.010	-0.3562	-0.1865	-0.2432	-0.2896	-0.3094
0.015	-0.3009	-0.1430	-0.1885	-0.2297	-0.2489
0.020	-0.2554	-0.1187	-0.1559	-0.1909	-0.2076
0.025	-0.2217	-0.1033	-0.1346	-0.1645	-0.1791
0.030	-0.1965	-0.0927	-0.1196	-0.1457	-0.1586
0.035	-0.1771	-0.0849	-0.1085	-0.1316	-0.1430

0.040	-0.1618	-0.0789	-0.0999	-0.1207	-0.1310
0.045	-0.1495	-0.0742	-0.0932	-0.1119	-0.1213
0.050	-0.1393	-0.0704	-0.0876	-0.1048	-0.1134

Thus we consider the influence of the rheological parameters to the values of the critical time and denote it by $t'_{cr.}$ In Tables 5-6 the values of $t'_{cr.}$ are given under $\gamma_1 = 0.3$, $\gamma_2 = 0.3$, $E = 500$ with various ω and α'. It follows from these results that if $\in \to \in_{cr.0}$ then $t'_{cr.} \to 0$; if $\in \to \in_{cr.\infty}$ then $t'_{cr.} \to \infty$. Moreover these results show that the values of $t'_{cr.}$ increase monotonically with ω. However the values of $t'_{cr.}$ decrease with $|\alpha'|$. These results agree with the well-known mechanical considerations and using these results we can determine the long-time strength of the nanocomposites with CNT under theirs compression along the CNT.

Table 5. The values of $t'_{cr.}$ under $E = 500$, $\gamma_1 = 0.3$, $\gamma_2 = 0.3$ with various ω, α' and $\in$

$\in$	$\omega = 0.5$			$\omega = 1.0$		
	$\alpha' = -0.3$	$\alpha' = -0.5$	$\alpha' = -0.7$	$\alpha' = -0.3$	$\alpha' = -0.5$	$\alpha' = -0.7$
-0.0520	0.1011	0.0533	0.0120	0.1303	0.0761	0.0217
-0.0530	0.0731	0.0339	0.0057	0.0891	0.0447	0.0090
-0.0540	0.0510	0.0205	0.0025	0.0594	0.0254	0.0035
-0.0550	0.0338	0.0115	0.0010	0.0378	0.0135	0.0013
-0.0560	0.0205	0.0057	0.0003	0.0222	0.0064	0.0004
-0.0570	0.0105	0.0023	0.0001	0.0110	0.0024	0.0001
-0.0580	0.0035	0.0005	0.0001	0.0036	0.0005	0.0001

Table 6. The values of $t'_{cr.}$ under $E = 500$, $\gamma_1 = 0.3$, $\gamma_2 = 0.3$ with various ω, α' and $\in$

$\in$	$\omega = 2.0$			$\omega = 3.0$		
	$\alpha = -0.3$	$\alpha = -0.5$	$\alpha = -0.7$	$\alpha = -0.3$	$\alpha = -0.5$	$\alpha = -0.7$
-0.0520	0.2641	0.2046	0.1127	1.1343	1.5741	3.3812
-0.0530	0.1480	0.0910	0.0292	0.3274	0.2764	0.1861
-0.0540	0.0855	0.0422	0.0082	0.1396	0.0838	0.0255
-0.0550	0.0489	0.0193	0.0022	0.0667	0.0298	0.0046
-0.0560	0.0263	0.0081	0.0006	0.0319	0.0106	0.0009
-0.0570	0.0122	0.0028	0.0001	0.0136	0.0033	0.0002
-0.0580	0.0038	0.0006	0.0001	0.0039	0.0006	0.0001

Conclusions

In this study, within the framework of the piecewise homogeneous body model with the use of the Three-Dimensional Geometrically Nonlinear Exact Equations of the Theory of Viscoelasticity the approach for the investigation of the internal stability loss (microbuckling) in the structure of the viscoelastic unidirected fibrous nanocomposites under compression along the fibers is developed. This development concerns mainly the case where the viscoelastic matrix contains a periodical curved carbon nanotube (CNT) and It is assumed that the CNT have an initial infinitesimal imperfection. The form of this imperfection is taken as periodical curving of the CNT. For the stability of the rising of the initial imperfection with the time under fixed

external compressed forces the Three-Dimensional Geometrically Nonlinear Exact Equations of the Theory of Viscoelasticity is employed. Introducing the dimensionless small parameter characterizing the degree of the insignificant initial imperfection for the solution to the corresponding nonlinear boundary value problem, the perturbation of the boundary-shape method is employed. It is proven that the equations and relations related to the first and subsequent approximations are the corresponding equations and relations of the TDLTS. For each approximation the corresponding closed system of linearized equations and contact conditions are obtained and for the solution of these equations the Laplace transformation with respect to time and method of separation of variables are employed. For determination of inverse Laplace transform the Schapery method is used. It is proven that the values of the critical parameters can be determined in the framework of the zeroth and first approximations only.
Using the developed approach the numerical results related to the critical time are also analyzed. It follows from these results that if $\in \to \in_{cr.0}$ then $t'_{cr.} \to 0$; if $\in \to \in_{cr.\infty}$ then $t'_{cr.} \to \infty$. Moreover these results show that the values of $t'_{cr.}$ increase monotonically with ω. However the values of $t'_{cr.}$ decrease with $|\alpha'|$. There is non-monotonic character between the parameters γ_1 and $\in_{cr,0}$, $\in_{cr,\infty}$, but the values of $\in_{cr,0}$ and $\in_{cr,\infty}$ decrease with γ_2. However the values of $\in_{cr,\infty}$ increase with ω. The values of $\in_{cr,0}$ and $\in_{cr,\infty}$ decrease with *E*..

Acknowledgements

This research has been supported by Yıldız Technical University Scientific Research Projects Coordination Department. Project Number: 2014-07-03-DOP01.

References

[1]. Guz, A.N., Mechanics of compressive failure of composite materials. Kiev: Naukova Dumka; 1990.
[2] Guz, A.,N. and Lapusta, Yu.N., Three-dimensional problems of the near-surface instability of fiber composites in compression (model of a piecewise-uniform medium) (survey). Inter. Appl. Mech. 1999; 35, No7, 641-671.
[3. Guz, A.N., Dekret, V.A. and Kokhanenko Yu.V., Plane problems of stability of composite materials with a finite size filler. Mechan. Comp. Materials 2000; 36, No1, 77-86.
[4] Akbarov, S.D., Sisman, T. and Yahnioglu, N. On the fracture of the unidirectional composites in compression. Int.J.Engng Sci 1997; 35, No 12/13, 1115-1136.
[5] Akbarov, S.D., Kosker, R.,. Fiber buckling in a viscoelastic matrix. Mechanics of Composite Materials 2001; 37(4), 299-306.
[6] Akbarov, S.D., Kosker, R.,. Internal Stability Loss of Two Neighbouring Fibers in a Viscoelastic Matrix. International Journal of Engineering Science 2004; 42, 1847–1873.
[7]. Akbarov, S.D. Stability Loss and Buckling Delemination:Three-Dimensional Linearized Approach for Elastic and Viscoelstic Composites. Berlin: Springer: 2013.
[8] .Akbarov, S.D., Microbuckling of a Double-Walled Carbon Nanotube Embedded in an

Elastic Matrix, International Journal of Solids and Structures 2013; 50, 2584- 2596.
[9] Akbarov, S.D. and Guz, A.N., Mechanics of Curved Composites, Kluwer Academic Publishers: 2000.
[10] Rabotnov, Yu., N., Elements of Hereditary Mechanics of Solid Bodies. Moscow :Nauka; 1977.
[11] Schapery, R.A., Approximate Methods of Transform inversion For Viscoelastic Stress Analyses. Proc. US. Natl. Cong. Appl. ASME 1966; 4, 1075-1085.