Probabilistic Analysis of Load Flows in DC Power Systems

Rıfkı Terzioğlu and T. Fedai Çavuş
Faculty of Engineering, Department of Electrical and Electronics Engineering, Sakarya University, Turkey

Abstract

This paper discusses a technique which the DC power flow problem in a power system is analyzed probabilistically. There are some uncertainties that deterministic methods avoid which are important in system planning and security. The aim of the paper is to take the uncertainties of the power data into account while the topology of the network is kept constant. The expected values and standard deviation of each lines power flow is calculated. The sensitivity coefficients are calculated for each line and have great importance because the sensitivity coefficients help us knowing how changes at nodal data will influence the power flows in each line. This technique calculates all the possible power flows and their probabilities of occurrence. In addition the probabilistic method is implemented on a sample system and it is shown that the probabilistic analysis gives much wider information on the power flow problem then the deterministic analysis.

Key words: Probabilistic load flow, Load distribution, Power systems, Probability

1. Introduction

Power flow studies are a major tool to see the current and future performance of the power system. It is being used in planning and operating power systems for several years. There are very well developed deterministic methods that are used in power flow studies that permit them to be made very quickly, accurately and efficiently. In these deterministic methods the nodal loads, generation and topology of the network is kept constant. In practice however the data of the nodal loads and generation can only be known with limited accuracy. In this reason the PLF (Probabilistic Load Flow) was proposed by Borkowska for evaluation of the power flow considering uncertainties. The uncertainties encountered may be due to;

- measurement error,
- forecast inaccuracy or,
- outages of system elements [1].

The main purpose and reason of using probabilistic analysis instead of deterministic is to consider the uncertainties and model these statistical variations in the input data of the power system.

A number of papers have been published that have modeled the load flow problem probabilistically [1]-[5]. In this paper the probabilistic method has been explained.

*Corresponding author: Address: Faculty of Engineering, Department of Electrical and Electronics Engineering Sakarya University, 54187, Sakarya TURKEY. **E-mail address: rterzioglu@sakarya.edu.tr, Phone: +902642955789**

2. Formulation of the DC Load-Flow Problem

DC power flow is a simplification, and linearization of a full AC power flow. DC power flow looks only at active power flows, neglecting voltage support, reactive power management and transmission losses. Thanks to its simplicity and linearity it is very often used for contingency analysis and techno-economic studies of power systems for assessing the influence of commercial energy exchanges on active power flows in the transmission network [6]. The detailed form of the active power dc load flow equations are:

$$P_i = V_i \sum_{k=1}^{n} V_k (G_{ij} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}) \quad \textbf{(1)}$$

$$P_{ik} = -t_{ik} G_{ik} V_i^2 + V_i V_k (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}) \quad \textbf{(2)}$$

where B_{ik} is the imaginary part of element *ik* of admittance matrix, G_{ik} is the real part of element *ik* of the admittance matrix, n is the number of nodes, P_i is the injected active power at node *i*, P_{ik} is the active power flow in line *i*-*k*, t_{ik} is the transformer tap ratio, V_i is the voltage magnitude at node *i*, δ_i is the angle at node *i* referred to slack node and δ_{ik} is the difference in angles between nodes *i* and *k*.

In order to reduce calculation time, the power flow problem can be simplified in by making the system linear. A number of assumptions are made in order to make his linearization practicable. These assumptions are:

(a) branch flows are linearly related to net nodal powers and all nodal voltages are equal,
(b) active and reactive power flows are independent,
(c) balancing of power is restricted to the slack node and the losses in the system are neglected,
(d) the topology of the network is constant [2].

The problem with the deterministic technique is that the solution can only be accurate as the input data. Any variation of the input data can cause changes in the load flow solution. To consider these changes in the load flow solution the problem can be modeled probabilistically.

3. The Probabilistic Model

As discussed in Section 2, the nodal data for the system can be considered as random variables. The nodal loads and generation are defined as random variables because of factors like the change in the load demand and the generator outages. The traditional deterministic DC load flow only finds line flows under a specified operating condition. On the other hand, the probabilistic load flow takes the uncertainties into considerations, such as the probability of a line flow being greater than its thermal rating under load uncertainties and random contingencies.

The nodal data are specified in terms of probability density functions. There are many analytical models for defining density functions in power system analysis. In this paper these are normal distributions for representing nodal-load estimate, binomial distributions for representing a set of identical generator units, discrete variables when neither of the previous distributions are suitable and one point values when a power has a unity probability of occurrence [2].

All the input data are first converted to the expected values (μ) and their variance (σ^2) by their distributions, e.g.; in normal distribution: μ and σ (standard deviation) are given; in binomial distribution:

$\mu=n(1-q)R$ and $\sigma^2=nq(1-q)R^2$,

where n=number of identical units, q=outage coefficient of each unit and R=power rating of each unit and in discrete values:

$$\mu=\sum_{i=1}^{m} x_i p_i \quad \text{and} \quad \sigma^2=\sum_{i=1}^{m}(x_i-\mu)^2 p_i$$

where m=number of discrete values and x_i is the ith discrete value having a probability of occurrence of p_i. [2]

Consider Eq. 1 assuming $V_i=V_k=1p.u.$, $G_{ik}=0$ (zero line resistance) and $\sin\delta_{ik} \approx \delta_{ik}$, we obtain,

$$P_i=\frac{1}{X_{ik}}\delta_{ik} \tag{3}$$

where X_{ik} is the reactance of the line joining buses i and k. The above equation in matrix form and inverting gives;

$$\delta=Y^{-1}P \tag{4}$$

where $Y_{ik}=-1/X_{ik}$ and $Y_{ii}=\sum_{i\neq k} 1/X_{ik}$, in which the slack bus row and column are deleted. This equation is known as a dc form of the load flow problem. Under these circumstances Eq. 2 becomes,

$$P_{ik}=\frac{\delta_i-\delta_k}{X_{ik}}=\sum_j H_{(ik)j}P_j \tag{5}$$

The H matrix contains network distribution factors. It is called the sensitivity coefficients and the notation "(ik)j" represents the amount of real power flowing in line l (line between buses i and k) as a result of injection of 1MW at bus j. If node j is slack, $H_{(ik)j}=0$ [7]. The calculation of the sensitivity coefficients only needs network data and doesn't require any nodal data. These coefficients can be useful. They show the change in the power flow in each line owing to a specified nodal power change [2].

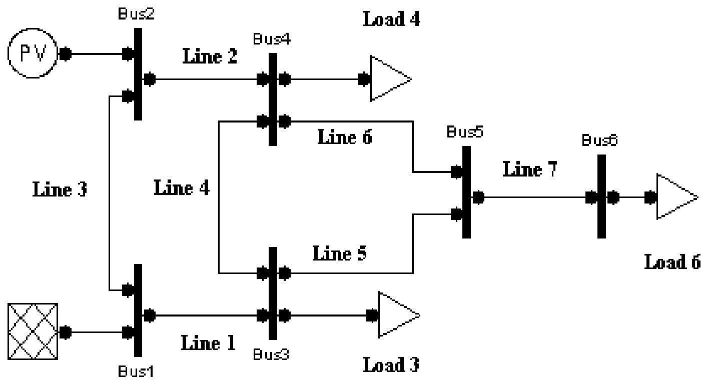

Figure 1. Sample System

4. Data and Results

To exemplify the probabilistic approach, consider the sample system shown in Figure 1. This sample system has 5 nodes that are independent, one reference node (node 1) and 7 lines. The nodal data used are shown in Table 1 and the network data are shown in Table 2. The nodal data contains binomial and discrete distributions.

Table 1. Nodal Data Used for the Sample System

Node	**Probability Function**	**Number of Units**	**Power (MW)**	
				Outage Coefficient
1	Binomial	10	10	0.0148
2	Binomial	10	9	0.12
				Probability of Occurrence
3	Discrete		-60	0.2
			-63	0.2
			-65	0.2
			-68	0.2
			-70	0.2
4	Discrete		-24	0.1
			-26	0.15
			-30	0.5
			-34	0.15
			-36	0.1
5	Discrete		-74.7	0.15
			-76.7	0.45
			-80	0.2
			-82.8	0.1
			-85	0.1

The expected values and standard deviation of the power flow results in each line is shown in Table 3. The deterministic values were assumed to be expected values. In the deterministic solution only the expected flows were obtained and the solution includes no information about the standard deviation. The standard deviation of the power flows in each line is usually low about %3-4. Standard deviation of the line between nodes 1 and 2 is the highest value in this sample and its value is %17,964. The reason the standard deviations are small is that the input nodal data's standard deviations were small values. If we had kept the expected value of the input nodal data constant and increase the standard deviation, the expected values of the power flows in each line would have been the same although the standard deviation would increase and the probability density functions of the power flows would have changed. It should be known that the five highest occurrence values of the binomial distribution have been taken in the case.

Table 2. Network Data for System Shown in Figure 1

Line Number	**Node Number**		**Reactance (Ω)**	**Susceptance ($x10^{-3}$)**
	Sending End	Receiving End		
1	1	3	15	0.05
2	2	4	8	0.15
3	1	2	5	0.25
4	3	4	10	0.075
5	3	5	16	0.075
6	4	5	16	0.075
7	5	6	10	0.075

Table 3. Sample System Power Flow Results

Node i-j	**Deterministic Power Flow (MW)**	**Probabilistic Power Flow (MW)**	**Standard Deviation (%)**
1-3	75	74.505	3.7542
2-4	99.547	98.995	3.1629
1-2	20.388	19.795	17.964
3-4	-22.668	-22.663	6.5029
3-5	32.442	32.168	5.0904
4-5	46.61	46.332	3.6225
5-6	78.682	78.5	4.0581

Table 4. Sensitivity Coefficients for the Line between Nodes 1 and 2

Due to Node	**2**	**3**	**4**	**5**	**6**
Sensitivity Coefficient	-0.8623	-0.4248	-0.6393	-0.5333	-0.5337

The sensitivity coefficients for the line between nodes 1 and 2 are shown in Table 4. Tables of this type give the information how changes in the nodal data will affect the power flows in each line.

The density functions of line 1 and 2 are shown in Fig. 2.1 and Fig. 2.2. These density functions show that the change in the input data affects the power flows in each line. It is seen from the figures that the probability of expected value is not always the greatest. In Fig.2.2 it is clearly seen that the value of the power flow between nodes 2 and 4 has a probability of %1 to be higher than 106,5 MW and %5 probability to be higher than 105 MW. This fact should be known when a power system is being planned and operated. This kind of information is in great importance in situations where security and reliability is very important.

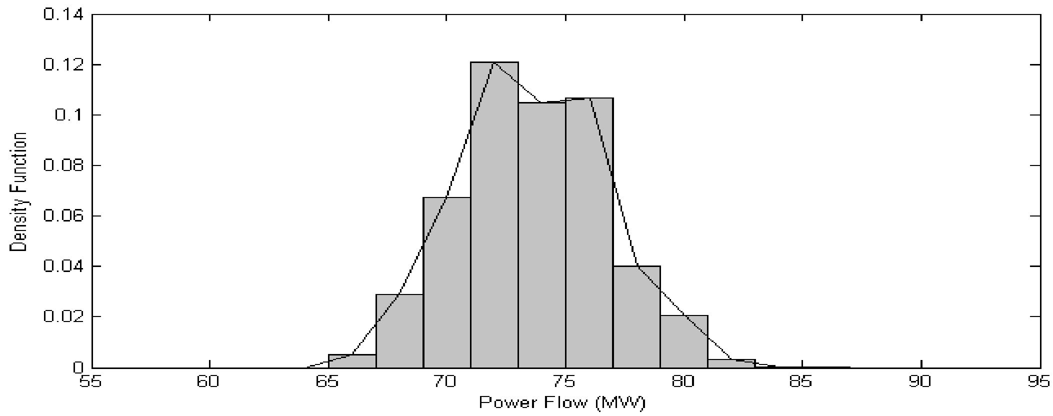

Figure 2.1. Power Flow between Nodes 1 and 3

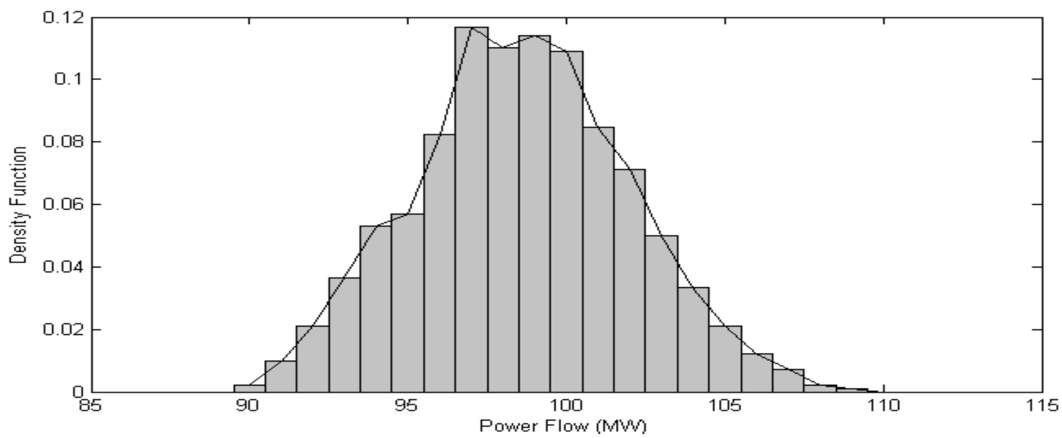

Figure 2.2. Power Flow between Nodes 2 and 4

5. Conclusions

The previous sections have shown the wide information that can be gained by the PLF. Although the expected value of the power flows calculated in probabilistic method is not exact as the deterministic method, several advantages appear. The probable power flows were calculated with their probability. This method objectively solves the uncertainties problem in the load flow problem and considers all the possibilities like generator outages and load changes. It helps to think about the worst conditions that can happen with the wider information the density functions give. Applying probabilistic studies in power systems allow gaining information for the future conditions.

6. Acknowledgements

Two of the authors, T.F. Çavuş and R. Terzioğlu are grateful to the Department of Electrical and Electronics Engineering, Sakarya University for providing the facilities to conduct this study. The author R. Terzioğlu is grateful to İbrahim Aydın for his helps and ideas on the research.

7. References

[1] Borkowska, B.,"Probabilistic Load Flow", IEEE Trans. Power Apparatus and Systems, vol. PAS-93, no.3, pp 752-755, 1974.

[2] Allan, R.N., Borkowska, B. and Grigg, C.H. "Probabilistic Analysis of Power Flows", Proceedings of the Institution of Electrical Engineers, vol.121, no.12, pp.1551-1556,1974.

[3] Allan, R.N., Grigg, C.H. and Al-Shakarchi, M.R.G., "Numerical Techniques in Probabilistic Load Flow Problems", International Journal for Numerical Methods in Engineering, vol. 10, pp. 853-860, 1976.

[4] Allan, R.N., Grigg, C.H., Newey, D.A. and Simmons, R.F., "Probabilistic Power-Flow Techniques Extended and Applied to Oğerational Decision Making", Proceedings of the Institution of Electrical Engineers, vol. 123, no. 12, pp. 1317-1324, 1976.

[5] Allan, R.N., Al-Shakarchi, M.R.G., "Probabilistic AC Load Flow", Proc. IEE, 1976, 123, (6), pp. 531-536.

[6] D. Van Hertem, J. Verboomen, K. Purchala, R. Belmans, W. L. Kling, "Usefulness of DC Power Flow for Active Power Flow Analysis with Flow Controlling Devices", AC and DC Power Transmission, The 8th IEE International Conference, 2006.

[7] George J. Anders, "Probability Concepts in Electric Power Systems", New York, John Wiley & Sons, Inc. 1990.