Control of Harmonic Patterns Using Shifted Variable Pulse Amplitude Excitation Strategy in 4D Arrays

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Abstract

Optimization techniques may be used in time-modulated arrays in order to shape the radiation pattern. Optimized excitation strategies allow the radiation pattern to be steered into desired direction while the side lobe levels are being controlled. In this paper, variable pulse amplitude strategy is used with modifications and differential evolution algorithm is applied to obtain the best results. Using this approach, individual harmonic levels may be enhanced or suppressed in order to be used in different applications. Furthermore, an explanatory example is given in order to visualize the results.

Key words: Differential Evolution, optimization, TMLA, harmonic beam steering

1. Introduction

After the concept so called time modulation was introduced in late 50’s, it started to become hot topic again since 2002 [1,2]. High speed RF switches are being used to switch antenna elements with respect to an excitation scheme which is periodic in time. The switching process, naturally, causes infinite number of harmonic radiations which are called as sideband radiations whose frequencies are related to multiplies of the switching frequency. On one hand, sideband radiations are thought to be power loss and are tried to be calculated [3-8] and suppressed with different methods such as Differential Evolution (DE) or Particle Swarm Optimization (PSO) [9-10]. On the other hand, instead of suppressing the sidebands, it is asserted in many studies such as [11-15] that sideband radiations may be used in different applications like direction finding, secure communication and communication over sidebands.

In literature, different excitation schemes such as variable aperture size, pulse shift, pulse split and variable pulse amplitude are used [2,13,16-17] but in this study a new approach which combines pulse shift and variable aperture size are proposed to have flexibility in optimization.

This paper organized as follows: time modulation background and theoretical calculations of proposed excitation scheme is given in the next section. Third section consist of design and optimization parameters according to the selected algorithm and fourth section presents the results of an explanatory design with discussion. Finally, the paper is concluded in conclusion section and in order not to disrupt the integrity, complex and long equations are referred to appendices.

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2. Time Modulation

The conventional array factor of $N$ isotropic elements equally spaced with $d = 0.5\lambda$ and located along $z$-axis may be expressed as:

\[ AF(\theta, t) = \sum_{n=1}^{N} I_n e^{j\beta_n} e^{jkz_n \cos \theta}, \]  

(1)

where $I_n$, $\beta_n$ and $z_n$ stands for excitation amplitude, excitation phase and distance of $n^{th}$ element to the system origin, respectively. $\theta$ represents elevation angle according to $z$-axis and $k = 2\pi/\lambda$ denotes the wavenumber. If each element of the array is switched using high-speed RF switches, switching function affects the array factor and (1) may be re-written as:

\[ AF(\theta, t) = \sum_{n=1}^{N} I_n e^{j\beta_n} U_n(t) e^{jkz_n \cos \theta}. \]  

(2)

Here, $U_n(t)$ is the periodic switching function and any periodic function may be used in order to excite the elements. As every periodic function, $U_n(t)$ can also be decomposed into complex Fourier series which results infinite number of harmonic signals and the most general form of time modulated array factor may be expressed as:

\[ AF(\theta, t) = \sum_{m=-\infty}^{\infty} \sum_{n=1}^{N} I_n e^{j\beta_n} C_{m,n} e^{jkz_n \cos \theta} e^{j(w_0+m\omega_p)t}, \]  

(3)

where $C_{m,n}$, $w_0 = 2\pi f_0$ and $\omega_p = 2\pi f_p$ denotes the complex Fourier coefficient of $n^{th}$ element in $m^{th}$ harmonic level, the angular operating frequency and the angular switching frequency, respectively. It should be noted that these definitions are valid under far field conditions and it is assumed that $f_0 \gg f_p$.

The most common excitation strategies in literature use on-off switching, however, to handle some disadvantages such as array silencing, which means all elements are switched off, a different scheme namely variable pulse amplitude (VPA) proposing switching between an amplifier or an attenuator was introduced by Aksoy [13]. The most basic form of VPA may be expressed as:

\[ U_n(t) = \begin{cases}  K_{n}^1 , & 0 < t \leq t_n^1 \\ K_{n}^2 , & t_n^1 < t \leq T_p \end{cases}, \]  

(4)

where $K_{n}^1$, $K_{n}^2$ and $t_n^1$ denote relative pulse amplitudes of adjacent pulses and the amplitude reversal time instant for $n^{th}$ element, respectively. Difference between common on-off excitation strategies
and VPA can be clearly seen in Fig. 1.

**Figure 1.** (a) Common on-off switching scheme of two elements (b) VPA switching scheme for two elements

Shifting the pulses in any type of excitation strategy directly brings pulse starting instants as a complex exponential in harmonic frequencies [16]. In this study, we introduce a new excitation strategy namely “Shifted-VPA” which combines VPA and pulse split methods in order to control radiation patterns. Since array silencing may occur due to shifted areas, we used additional pulse offset value, $K_n^0$, instead of switching off the element and proposed excitation strategy may be defined as:

$$U_n(t) = \begin{cases} K_n^1, & t_n^1 < t \leq t_n^2 \\ K_n^2, & t_n^2 < t \leq t_n^3 \\ K_n^0, & \text{otherwise} \end{cases}$$

Here, $t_n^1$, $t_n^2$ and $t_n^3$ stand for starting instant of first pulse, amplitude reversal time instant and finishing instant of second pulse, respectively. In Fig. 2, time domain diagram of Shifted-VPA may be seen.

**Figure 2.** Time domain representation for Shifted VPA Scheme
Using (5), complex Fourier coefficients for Shifted-VPA scheme can be calculated and in order not to disrupt integrity details of the calculations are presented in Appendix A. Using (A.4) and (A.9), $C_{m,n}$ values may be written as:

$$C_{m,n} = \begin{cases} K_n^0 + \sum_{q=1}^{3} \Delta_n^q \tau_n^q, & m = 0 \\ - \frac{1}{m\pi} \left( \frac{K_n^0}{2j} + \sum_{q=1}^{3} \Delta_n^q e^{-j m \pi \tau_n^q} \sin(m \pi \tau_n^q) \right), & |m| \geq 1 \end{cases}$$  

Here, $\Delta_n^{q+1} = K_n^q - K_n^{(q+1)(mod \ 3)}$ and $\tau_n^{q+1} = \tau_n^q / T_p$ where $q \in \{0,1,2\}$ represent normalized pulse difference of contiguous pulses and amplitude reversal time instants with respect to switching period, respectively.

3. Array Design and Optimization

Let us consider a 20 element array whose identical, zero-phased ($\beta_n = 0$) elements are located along z-axis with $d = 0.45\lambda$ spacing. Excitation amplitude for all elements are selected equal to unity (i.e. $I_n = 1$) and pulse offset value is set to 0.3 (i.e. $K_n^0 = 0.3$). In order to control radiation patterns, DE using “DE/best/1/bin” scheme is applied with mutation factor, crossover factor and population size of 0.65, 0.90 and 150, respectively.

As it can be seen in (6), $\Delta_n^q$ and $\tau_n^q$ where $q \in \{0,1,2\}$ may be optimization parameters. The six parameter is used for optimization and the cost function is calculated according to:

- Steer main radiation and fundamental harmonic radiation to broadside and 60°, respectively
- Reduce side lobe level (SLL) for both radiations below -20dB.
- Suppress sideband radiations except for the first harmonic radiation.

No constraint are applied to fundamental radiation main beam and an iteration limit is set as the stop criterion (i.e. 150 iterations). The hardware of PC which is used for optimization process consist of Intel i7 M640 2.80GHz CPU, 8GB RAM and 128GB SSD. In addition, Windows 10 and MATLAB R2016b are the software setup of optimization PC.

4. Results and Discussion

Optimized excitation scheme is being shown in Fig. 3 and optimized normalized pulse difference values are given Table 1. Furthermore, according to optimization results in Fig. 4 and Fig. 5 shaped radiation patterns for main and fundamental harmonic radiations and maximum levels of first ten
harmonic level after 150 iterations is presented, respectively. The iteration takes average of 116.86 seconds and once the results are examined, SLLs for main and fundamental radiations and SR levels stay within acceptable limits ($<-20dB$) while first harmonic radiation is successfully steered to $60^\circ$.

**Table 1.** Optimized Normalized Pulse Difference Values ($\Delta_n^h$)

<table>
<thead>
<tr>
<th>n</th>
<th>$\Delta_n^1$</th>
<th>$\Delta_n^2$</th>
<th>$\Delta_n^3$</th>
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<tr>
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<tr>
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After 40 experiment, it is understood that results are repeatable and reside in acceptable level. On one hand, with these four constraints and six optimization parameters, iteration counts should be higher than 150 in order to have more accurate results but elapsed time for higher number of iterations are dramatically increases. On the other hand, instead of optimizing six parameter, pulse difference values, $\Delta n^q$, may be fixed to an optimal value and optimizing only pulse starting/finishing instants, $\tau n^q$, should decrease total time and enables having nearly same results within less iterations but this is out of scope for this study and assigned as future work.

Conclusions

The study introduces a modification to variable pulse amplitude excitation strategy and examines the effects of shifting pulses to radiation patterns. It is concluded that, with basic optimization algorithms, radiation patterns are steerable in order to be used in different applications such as secure communication and communication over sidebands. Even with lower iteration numbers, resultant side lobe levels and unwanted sideband radiations reside in acceptable level once the criterions of telecommunication authorities are considered for most applications ($< -20dB$).
Appendix A

In this appendix, in order to maintain paper’s integrity, steps for calculating complex Fourier coefficients of Shifted-VPA is being shown. Complex Fourier coefficients are calculated with:

\[ C_{m,n} = \frac{1}{T_p} \int_{-\infty}^{\infty} U_n(t)e^{-j\omega_p t} dt. \]  

(A.1)

Using (5), for \( m = 0 \):

\[ C_{m,n} = \frac{1}{T_p} \left[ \int_0^{t_1} K_n^0 dt + \int_{t_1}^{t_2} K_n^1 dt + \int_{t_2}^{t_3} K_n^2 dt + \int_{t_3}^{T_p} K_n^0 dt \right]. \]  

(A.2)

\[ C_{m,n} = \frac{1}{T_p} \left[ K_n^0 t \left| _0^{t_1} + K_n^1 t \left| _{t_1}^{t_2} + K_n^2 t \left| _{t_2}^{t_3} + K_n^0 t \left| _{t_3}^{T_p} \right. \right. \right. \right. \]. \]  

(A.3)

\[ C_{m,n} = \tau_n^1 (K_n^0 - K_n^1) + \tau_n^2 (K_n^1 - K_n^2) + \tau_n^3 (K_n^2 - K_n^0) + K_n^0 \]
\[ = \Delta_n^1 \tau_n^1 + \Delta_n^2 \tau_n^2 + \Delta_n^3 \tau_n^3 + K_n^0 \]  

(A.4)
For $|m| > 0$:

$$C_{m,n} = \frac{1}{T_p} \left[ \int_{t_n}^{t_n^1} K_0^0 e^{-jm\omega pt} dt + \int_{t_n^1}^{t_n^2} K_1^0 e^{-jm\omega pt} dt + \int_{t_n^2}^{t_n^3} K_2^0 e^{-jm\omega pt} dt + \int_{t_n^3}^{\tau_p} K_n^0 e^{-jm\omega pt} dt \right]. \quad (A.5)$$

$$C_{m,n} = -\frac{1}{jm\omega_p T_p} \left[ K_n^0 e^{-jm\omega pt} \left| \begin{array}{c} t_n^1 \\ t_n^1 \\ t_n^2 \\ t_n^1 \\ t_n^1 \\ t_n^2 \\ t_n^3 \\ t_n^3 \\ t_n^3 \\ t_n^3 \end{array} \right| + K_n^0 e^{-jm\omega pt} \right] \left| \begin{array}{c} T_p \\ t_n^3 \\ t_n^3 \\ t_n^3 \end{array} \right|, \quad \omega_p T_p = 2\pi. \quad (A.6)$$

$$C_{m,n} = -\frac{1}{jm2\pi} \left[ (K_0^0 - K_1^0) e^{-jm2\pi \tau_n^1} + (K_1^0 - K_2^0) e^{-jm2\pi \tau_n^2} \\ + (K_2^0 - K_n^0) e^{-jm2\pi \tau_n^3} + K_n^0 \right]. \quad (A.7)$$

$$C_{m,n} = -\frac{1}{jm2\pi} \left[ \Delta_1^1 e^{-jm2\pi \tau_n^1} + \Delta_2^1 e^{-jm2\pi \tau_n^2} + \Delta_3^1 e^{-jm2\pi \tau_n^3} + K_n^0 \right]. \quad (A.8)$$

After basic calculations, coefficients may be expressed as:

$$C_{m,n} = -\frac{1}{mn} \left[ \Delta_1^1 e^{-jm\pi \tau_n^1} \sin(m\pi \tau_n^1) + \Delta_2^2 e^{-jm\pi \tau_n^2} \sin(m\pi \tau_n^2) + \Delta_3^3 e^{-jm\pi \tau_n^3} \sin(m\pi \tau_n^3) + \frac{K_n^0}{2j} \right]. \quad (A.9)$$

References