

Damped Response of Cross-Ply Laminated Doubly Curved Shells

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Abstract

This paper aims to present an alternative analytical method for damped response of cross-ply laminated doubly-curved shells. In the proposed method, the governing equations of first order shear deformation laminated shell are obtained by Navier solution procedure. Materials of the shell laminates are assumed to be linear elastic or viscoelastic. Time-dependent equations are transformed to the Laplace domain and then Laplace parameter dependent equations are solved numerically. The results obtained in the Laplace domain are transformed to the time domain with the help of modified Durbin's numerical inverse Laplace transform method. Verification of the presented method is carried out by comparing the results with those obtained by Newmark method and ANSYS finite element software. The numerical results have proved that the presented procedure is a highly accurate and efficient solution method and it can be easily applied to the laminated composite shell problems

Key Words: Viscoelastic materials; Damped vibration; Inverse Laplace transform; Doublycurved laminated shells

1. Introduction

Laminated composite shells are being increasingly used in all fields of engineering because of their many advantageous properties. The most advantageous part of shell structures is their load-carrying ability due to combination of laminates and their curvature. Therefore, many papers have been published on this subject. Transient vibration analysis of shells made of layers has a primary importance in engineering design. It is necessary to have a full understanding of the behavior of laminated composite shells.

Many researchers have investigated the dynamic analysis of shell structures with linear elastic materials. Toh et al. [1] examined the transient stress response of an orthotropic laminated open cylindrical shell. They presented the solution analytically which includes both contact deformation and transverse shear. Gong et al. [2] presented a set of analytic solutions to predict the dynamic response of simply supported laminated shells. Wu et al. [3] formulated an asymptotic theory for dynamic analysis of doubly curved laminated shells within the framework of three-dimensional elasticity. Chun and Lam [4] investigated the free and forced vibration of laminated curved panels subjected to the triangular, explosive and step loadings. The normal mode superposition method is used in the forced vibration analysis. Prusty and Satsangi [5] described the transient response of composite stiffened plates and shells. The governing undamped equation of motion is obtained with finite element method and Newmark's method is used for the direct time integration. Swaddiwudhipong and Lui [6] used modified nine-node degenerated shell elements to investigate the elastic and elasto-plastic dynamic response of laminated composite plate and shell structures. In their study, Newmark's algorithm is used for the direct time integration. Krishnamurthy et al. [7] used the finite element methods and the classical Fourier series to obtain impact response of a laminated composite cylindrical shell. Equations are solved by means of Newmark's algorithm combined with a predictor-corrector scheme. Her and Liang [8] studied the composite laminate and shell structures subjected to low velocity impact by the

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ANSYS/LSDYNA finite element software. Park et al. [9] presented static and dynamic analysis of laminated plates and shells. The transverse shear stiffness was defined by an equilibrium approach. To determine the element stiffness matrix, the Quasi-Conforming Technique was used. Newmark-b method was used for time integration. Jung and Han [10] investigated the vibration analysis of functionally graded material and laminated composite structures, using a refined 8-node shell element that allows for the effects of transverse shear deformation and rotary inertia. For the study, the Newmark-b time integration method was adopted.

In this paper, the damped dynamic analysis of doubly curved laminated shells is examined theoretically with the Laplace transform. Governing equations of dynamic system are transformed efficiently to a static case with applying Laplace method [11-15]. For verification the numerical results obtained with presented procedure are compared with those obtained Newmark method and ANSYS finite element software. The results obtained with the suggested method are found to be in excellent agreement with those in the literature.

2. Theoretical formulation

Suppose that the shell is composed of *N* orthotropic layers of uniform thickness. Co-ordinate system of a doubly curved laminated shell is shown in Fig. 1. Here, an orthogonal curvilinear coordinate system is composed from ξ_1, ξ_2, ζ coordinates. ξ_1 and ξ_2 curves are lines of curvature on the mid-surface of the shell.



Figure 1. Coordinate and Geometry of a laminated doubly-curved shell

2.1. Kinematics of the shell

The displacements along the local coordinate axes ξ_1, ξ_2 , and ζ at any point in the FSDT thick shell are assumed as [16]

$$\begin{cases} u_1(\xi_1, \xi_2, \zeta, t) \\ u_2(\xi_1, \xi_2, \zeta, t) \\ u_3(\xi_1, \xi_2, \zeta, t) \end{cases} = \begin{cases} u_0(\xi_1, \xi_2, t) \\ v_0(\xi_1, \xi_2, t) \\ w_0(\xi_1, \xi_2, t) \end{cases} + \zeta \begin{cases} \emptyset_1(\xi_1, \xi_2, t) \\ \emptyset_2(\xi_1, \xi_2, t) \\ 0 \end{cases}$$
(1)

where (u_1, u_2, u_3) are the displacements of any point in the laminated shell, (u_0, v_0, w_0) are the displacements of any point on the mid-surface of the shell. $(\emptyset_1, \emptyset_2)$ are the rotations of the reference surface, $\zeta = 0$, about the ξ_2 - and ξ_1 - coordinate axes, respectively.

$$\begin{cases} \varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\\ \gamma_{23}\\ \gamma_{13} \end{cases} = \begin{cases} \varepsilon_1^{\circ}\\ \varepsilon_2^{\circ}\\ \gamma_{12}^{\circ}\\ \gamma_{23}\\ \gamma_{13} \end{cases} + \zeta \begin{cases} \kappa_1\\ \kappa_2\\ \kappa_3\\ 0\\ 0 \end{cases}$$
(2)

where

$$\begin{aligned}
\varepsilon_{1}^{0} &= \frac{1}{\alpha_{1}} \frac{\partial u_{0}}{\partial \xi_{1}} + \frac{w_{0}}{R_{1}}, \quad \varepsilon_{2}^{0} &= \frac{1}{\alpha_{2}} \frac{\partial v_{0}}{\partial \xi_{2}} + \frac{w_{0}}{R_{2}}, \quad \gamma_{12}^{0} &= \frac{1}{\alpha_{1}} \frac{\partial v_{0}}{\partial \xi_{1}} + \frac{1}{\alpha_{2}} \frac{\partial u_{0}}{\partial \xi_{2}} \\
\gamma_{23} &= \frac{1}{\alpha_{2}} \frac{\partial w_{0}}{\partial \xi_{2}} + \phi_{2} - \frac{v_{0}}{R_{2}}, \quad \gamma_{13} &= \frac{1}{\alpha_{1}} \frac{\partial w_{0}}{\partial \xi_{1}} + \phi_{1} - \frac{u_{0}}{R_{1}} \\
\kappa_{1} &= \frac{1}{\alpha_{1}} \frac{\partial \phi_{1}}{\partial \xi_{1}}, \quad \kappa_{2} &= \frac{1}{\alpha_{2}} \frac{\partial \phi_{2}}{\partial \xi_{2}}, \\
\kappa_{3} &= \frac{1}{\alpha_{1}} \frac{\partial \phi_{2}}{\partial \xi_{1}} + \frac{1}{\alpha_{2}} \frac{\partial \phi_{1}}{\partial \xi_{2}} + \frac{1}{2} \left(\frac{1}{R_{2}} - \frac{1}{R_{1}}\right) \left(\frac{1}{\alpha_{1}} \frac{\partial v_{0}}{\partial \xi_{1}} - \frac{1}{\alpha_{2}} \frac{\partial u_{0}}{\partial \xi_{2}}\right)
\end{aligned}$$
(3)

2.2. Constitutive equations

Composite shell layers stacked on each other with the principal material 1 axis of the *k*th layer is oriented at an angle $\theta^{(k)}$ from the shell x_1 coordinate in the counterclockwise sense and $x_3^{(k)} = \zeta$. The stress-strain relations of the *k*th orthotropic lamina in the shell coordinate system are given as

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \\ \tau_{23} \\ \tau_{13} \end{pmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}^{(k)} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{13} \end{pmatrix}$$
(4)

where $(\sigma_{11}, \sigma_{22}, \tau_{12}, \tau_{23}, \tau_{13})$ are the stresses, $(\varepsilon_{11}, \varepsilon_{22}, \gamma_{12}, \gamma_{23}, \gamma_{13})$ are the strains and \overline{Q}_{ij} are the transformed reduced stiffnesses.

Based on the FSDT shell, the stress resultants are given in a compact form as $(N_{12})^{(N_{12})}$

$$\begin{pmatrix}
N_{11} \\
N_{22} \\
N_{12} \\
M_{11} \\
M_{22} \\
M_{12}
\end{pmatrix} = \begin{bmatrix}
[A] & [B] \\
[B] & [D]
\end{bmatrix} \begin{pmatrix}
\varepsilon_1^{\circ} \\
\varepsilon_2^{\circ} \\
\gamma_{12}^{\circ} \\
\kappa_1 \\
\kappa_2 \\
\kappa_3
\end{pmatrix}$$
(5)

in which

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{N} \int_{\zeta_k}^{\zeta_{k+1}} \bar{Q}_{ij}^{(k)}(1, \zeta, \zeta^2) d\zeta \qquad (i, j = 1, 2, 6)$$
(6)

where ζ_k and ζ_{k+1} are the coordinates of the upper and lower surfaces of the k*th* layer. Transverse force resultants are defined as

$$\begin{cases} Q_2 \\ Q_1 \end{cases} = K_s[A_s]\{\gamma\}$$
(7)

where the parameter K_s is the shear correction factor. Here, K_s is taken as 5/6. [A_s] are defined by

$$(A_{sij}) = \sum_{k=1}^{N} \int_{\zeta_k}^{\zeta_{k+1}} \overline{Q}_{ij}^{(k)} d\zeta \qquad (i, j = 4, 5)$$
(8)

3. Dynamic Solution procedure

The simply supported boundary conditions (SS-1) of cross-ply laminated doubly-curved shell base on FSDT are:

$$u_{1}(x_{1},0,t)=0, \quad u_{1}(x_{1},b,t)=0, \quad u_{2}(0, x_{2},t)=0, \quad u_{2}(a, x_{2},t)=0, \\ N_{1}(0, x_{2},t)=0, \quad N_{1}(a, x_{2},t)=0, \quad N_{2}(x_{1},0,t)=0, \quad N_{2}(x_{1},b,t)=0, \\ \phi_{1}(x_{1},0,t)=0, \quad \phi_{1}(x_{1},b,t)=0, \quad \phi_{2}(0, x_{2},t)=0, \quad \phi_{2}(a, x_{2},t)=0, \\ u_{3}(x_{1},0,t)=0, \quad u_{3}(x_{1},b,t)=0, \quad u_{3}(0, x_{2},t)=0, \quad M_{2}(x_{1},b,t)=0, \\ M_{1}(0, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \quad M_{2}(x_{1},0,t)=0, \\ M_{1}(0, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \quad M_{2}(x_{1},b,t)=0, \\ M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \\ M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \\ M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \\ M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \\ M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \\ M_{1}(a, x_{2},t)=0, \quad M_{1}(a, x_{2},t)=0, \quad$$

The boundary conditions in (9) are satisfied by the following expansions of generalized displacement field:

$$\begin{cases} u_{0}(x_{1}, x_{2}, t) \\ v_{0}(x_{1}, x_{2}, t) \\ w_{0}(x_{1}, x_{2}, t) \\ \phi_{1}(x_{1}, x_{2}, t) \\ \phi_{2}(x_{1}, x_{2}, t) \end{cases} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left\{ \begin{matrix} U_{mn}(t) \cos \alpha x_{1} \sin \beta x_{2} \\ V_{mn}(t) \sin \alpha x_{1} \cos \beta x_{2} \\ W_{mn}(t) \sin \alpha x_{1} \sin \beta x_{2} \\ X_{mn}(t) \cos \alpha x_{1} \sin \beta x_{2} \\ Y_{mn}(t) \sin \alpha x_{1} \cos \beta x_{2} \end{matrix} \right\}$$
(10)
where $\alpha = m\pi/a$, $\beta = n\pi/b$.

The mechanical loads are also expanded in double Fourier sine series

$$q(x_1, x_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Q_{mn}(t) \sin \alpha x_1 \sin \beta x_2$$
(11)

where

$$Q_{mn}(t) = \frac{4}{ab} \int_0^a \int_0^b q(x_1, x_2, t) \sin \alpha x_1 \sin \beta y \, dx_1 dx_2$$
(12)
System of equations of motion of ESDT shall for simply supported cross ply laminated shall

System of equations of motion of FSDT shell for simply supported cross-ply laminated shells are obtained using Hamilton's principle. Five partially differential equations can be obtained in terms of mid-plane surface displacement (u_0 , v_0 , w_0 , \emptyset_1 , \emptyset_2) by substituting the force and moment resultants from equations (5), (7) for the system equations of motion. Substituting the expansions (10) and (11) for the five partially differential equations yields the equations

$$[M_{mn}]_{5\times5} \{ \ddot{\Delta}_{mn} \}_{5\times1} + [K_{mn}]_{5\times5} \{ \Delta_{mn} \}_{5\times1} = \{ F_{mn} \}_{5\times1}$$
(13)
here $(\Delta_{mn}) = (U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn})$ is the displacement vector.

When we apply Laplace transform to Eq. (13) the following equation is obtained

$$[\overline{D}_{mn}]\{\overline{\Delta}_{mn}\} = \{\overline{F}_{mn}\} + \{\overline{F}_0\}$$
(14)

where, $\{\overline{\Delta}_{mn}\}\$ is the transformed displacement vector, $\{\overline{F}_{mn}\}\$ is the transformed external force vector, $\{\overline{F}_0\}\$ is the initial condition force vector and $[\overline{D}_{mn}]\$ is the transformed dynamic stiffness matrix. The Eq. (14), transformed time-independent problem in spatial coordinate is solved numerically by Gauss elimination.

 $[\overline{D}_{mn}]$ and $\{\overline{F}_0\}$ are given as

$$[\overline{D}_{mn}] = [K_{mn}] + z^2 [M_{mn}]$$

$$\{\overline{F}_0\} = z[M_{mn}] \{\Delta(0)\} + [M_{mn}] \{\dot{\Delta}(0)\}$$
(15)
(16)

where z is the parameter of Laplace transform. $\{\Delta(0)\}\$ is absolute initial displacement vector and $\{\dot{\Delta}(0)\}\$ is absolute initial velocity vector. Here, initial conditions are taken to be zero. The case of internal viscoelastic damping is treated with the help of the correspondence principle, as described, for example, in Boley and Weiner [17]. The correspondence principle can be stated as follows: Laplace transform of the viscoelastic solution can be obtained from

the Laplace transform of the elastic solution by replacing the elastic constants *E* and *G* by $E_v = E(1 + g z)$, $G_v = G(1 + g z)$ (17) respectively.

respectively.

4. Results and discussion

In the study, results are presented for simply supported (SS-1) orthotropic cross-ply laminated shells (a/b=1, a/h=10, R/a=10) based on FSDT subjected to suddenly-applied uniformly-distributed step load. The numerical results in the following example are obtained with the proposed method, by using Navier approach combined with direct time integration technique and by ANSYS finite element software. ANSYS software results of the laminated shells are obtained using (8x8) mesh scheme. The mid-point deflection and the normal stress at the center of shell results are obtained and illustrated in following figures. In this example, the deflection and normal stress ($\zeta = -h/2$) at the mid-point of the laminated shell are showed in the graphic forms. The normal stresses are calculated at the bottom surface ($\zeta = -h/2$) of the shells.

In this study, a distributed load with the amplitude $q_0=1000 \text{ N/m}^2$ is applied suddenly on the laminated shell. In the examples, the following material properties are used.

a=b=1 m., h=0.1 m., $R_1=R_2=R=10$ m. (a/b=1, R/a=10, a/h=10).

 $E_2=1 \times 10^9$ N/m2, $E_1=25E_2$, $G_{12}=G_{13}=0.5E_2$, $G_{23}=0.2E_2$, v=0.25, $\rho=2000$ kg/m³.

First, the accuracy of the method will be verified on an example. Verification of the presented method is carried out by comparing the results with those obtained by Navier procedure in conjunction with Newmark method and ANSYS finite element software. Shell with $(0^{\circ}/90^{\circ})$ layers is first analyzed to confirm the proposed method. To achieve the effect of time increment (*dt*), several Laplace transform parameters (*N*) and time increment values (*dt*) have been used. The mid-point deflection (*w*) and the normal stress (σ_y) for ($0^{\circ}/90^{\circ}$) laminates are presented in Figs. 2-3, respectively.



Figure 2. Vertical mid-point displacement versus time for $(0^{\circ}/90^{\circ})$ laminates



Fig. 2 shows that the time-varying values of mid-point deflection achieved by the suggested method are identical for different dt (0.00008, 0.00016, 0.00032, 0.00064) and N (64, 128, 256, 512). Similarly numerical results of σ_y at the bottom surface of the shell that obtained with various time increments are identical (Fig. 3). The mid-point deflections obtained with the aid of Navier solution combined with Newmark method and ANSYS finite element software results are presented in Fig. 4. Similarly the normal stresses are presented in Fig. 5.



Figure 4. Vertical mid-point displacement versus time for $(0^{\circ}/90^{\circ})$ laminates



Figure 5. Central stress versus time for $(0^{\circ}/90^{\circ})$ laminates

Fig. 4 and Fig. 5 show that the time increments of 0.00008 and finer had to be considered for consistent results. An exact match is obtained by using a coarse time increment of 0.00064 in the present method as opposed to much finer increment of 0.00008 in the Newmark time integration method. Generally, our method gives more accurate results when compared to the aforementioned step by step integration method.

Second, the materials of the shell laminates are assumed to be viscoelastic. In the viscoelastic case the Laplace transform of the viscoelastic solution can be obtained from the Laplace transform of the elastic solution by replacing the elastic constants. Damping can be incorporated very easily in the transformed domain. According to the correspondence principle the material constants are replaced with their complex counterparts in the Laplace domain. The mid-point deflection and the normal stress obtained with suggested method and ANSYS software are presented in Fig. 6 and Fig. 7, respectively. Figs. 6-7 show the results of elastic–dynamic and viscoelastic solution for various damping ratios (g=0.0001, 0.0003, 0.0008).



Figure 6. Vertical mid-point displacement versus time for $(0^{\circ}/90^{\circ})$ laminates



Fig. 6 and Fig. 7 show that the results of proposed method and ANSYS software agree with each other. In the elastic–dynamic case, the results of the shells oscillates about the static state. In the viscoelastic case, the results die out with time. The effect of the damping ratio is obvious; increasing the damping ratio causes the response to reach the static response much faster.

Generally, the present method gives more accurate results when compared to the aforementioned step by step integration method.

5. Conclusions

Based on the presented method the following conclusions can be drawn:

Application of Laplace transform reduces a dynamic problem to a static one, which can be solved numerically in the Laplace domain. In the Laplace domain one can incorporate damping effect very easily. In addition, it should be noted that natural frequencies and mode shapes are not needed in the solutions. The accuracy of the results of Newmark method depends on the time increment selection. The method proposed here, however, even with a coarse time increment gives highly accurate results. It is clear that the suggested procedure is much more efficient than the conventional step-by-step integration methods. The numerical results have proved that the present approach is a highly accurate and simple solution method and it can be easily applied to the shell problems.

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