

# Mechanical Component Design for Multiobjective Fuzzy Optimization by Using Genetic Algorithm

\*<sup>1</sup>Omer Kelesoglu, <sup>2</sup>Yuksel Esen <sup>3</sup>Cevdet Emin Ekinci <sup>\*1,2,3</sup>Faculty of Technology, Department of Civil Engineering, Firat University,23119, Elazig, Turkey

# Abstract

In this study, investigated the application of genetic algorithms for solving a multiobjective fuzzy optimization problem of mechanical component design. This method enables a flexible method for optimal system design by applying fuzzy objectives and fuzzy constraints. This paper presents a new design method for multiobjective mechanical component optimization problems. The conclude that genetic algorithms can produce good approximate solutions when applied to solve fuzzy optimization problems.

Key words: Welded beam, spring design, multiobjective fuzzy optimization, genetic algorithm

# **1. Introduction**

Characteristic to most engineering problems is that they are multiobjective. Usually there is no single solution for which all objectives are optimal. The solution to a multiobjective problem therefore comprises a set of solutions for which holds that there are no other solutions that are superior considering all objectives. These solutions are called Pareto-optimal. Hence, optimising a multiobjective problem is comprised of finding Pareto optimal solutions.

Zadeh first introduced the concept of fuzzy set theory [1]. Then Zimmermann applied the fuzzy set theory concept with some suitable membership functions to solve linear programming problem with several objective functions [2]. Rao et al. applied a fuzzy optimization technique with membership functions to solve the multiobjective engineering problem [3]. Recently, fuzzy theory and membership function values are widely used in engineering design. The membership function making is also a useful tool to solve the multiobjective design problems [4]. For such cases, the application of the fuzzy set theory is effective, and multiobjective fuzzy optimization techniques have been developed [5-7].

Genetic Algorithms were more fully developed after original work by Holland [8]. These approaches consist of optimization procedures based on principles inspired by natural evolution. Given a problem for which a closed-from solution is unknown, or impossible to obtain with classical methods, an initial randomly generated population of possible solutions is created. Its characteristics are then used in an equivalent string of genes or chromosomes that are later recombined with genes from other individuals. It can be shown that by using the natural selection process, the method gradually converges, towards the best-possible solution [9-10]. The GA manipulates a population of the potential solution for problems such as optimization. Genetic algorithms for multiobjective optimization problems have been proposed in the literature. They can be used to compute the membership functions of fuzzy sets [11-12]. Kiyota et.al have proposed a multiobjective fuzzy optimization method by genetic algorithm [13]. Since then, several papers have evolved that use genetic algorithms [14-16]. Therefore, genetic algorithms are theoretically and empirically have been proven to process robust search capabilities in complex spaces, thus offering a valid approach to problems requiring efficient and effective searching [17].

<sup>&</sup>lt;sup>1</sup> \*Corresponding author: Address: Faculty of Technology, Department of Civil Engineering Firat University, 23119, Elazig TURKEY. E-mail address: okelesoglu@firat.edu.tr, Phone: +905358498468

In this paper, the multiobjective fuzzy optimization design method by GA is applied to design of mechanical component. Specifically, the  $\lambda$ -formulation is combined with a genetic algorithm for solving multiobjective fuzzy optimization problems with design variables. Finally, the proposed method is applied to a simple design problem of a spring and a welded beam.

### 2. Multiobjective fuzzy optimization and membership function

In general, the mathematical model of multiobjective fuzzy optimization problem as;

$$\min f(x) = \{f_1(x), f_2(x), ..., f_i(x)\}^T$$
(1)

subject to the design constraints:

$$g_{j}^{(l)} \leq g_{j}(x) \leq g_{j}^{(u)} \qquad j = 1, 2, ..., m,$$
 (2)

Where x is the design vector, f(x) is the vector of objective functions,  $g_j(x)$  is the *j*th constraint function, and the superscripts *l* and *u* indicate the lower and upper bound values, respectively.

This max–min problem can be solved by using the fuzzy  $\lambda$ -formulation technique that can be stated mathematically as [3];

The membership function is written  $\mu_{fi}(x)$ :

$$\mu_{fi}(x) = \begin{pmatrix} 0, & if & f_i(x) > f_i^{\max} \\ \lambda_i, & if & f_i^{\min} < f_i(x) \le f_i^{\max} \\ 1, & if & f_i(x) \le f_i^{\min} \end{pmatrix}$$
(3)

and

$$\lambda_i = [f_i^{\max} - f_i(x)] / [f_i^{\max} - f_i^{\min}] \quad i = 1, 2, ..., k$$
(4)

The minimum and maximum possible values of the design criteria in the all continuous space are represented as  $f_i^{\min}$  and  $f_i^{\max}$  respectively.

Maximize 
$$\lambda$$
 (5)

subject to 
$$\lambda - \mu_{f_i}(x) \le 0, \quad i = 1, 2, ..., k,$$
 (6)

$$\lambda - \mu_{g_{i}}(x) \le 0, \quad j = 1, 2, ..., m,$$
(7)

$$x_i^l \le x_i^l \le x_i^u \quad j = 1, 2, ..., n.$$
 (8)

For solving the multiobjective programming problems, the following multiobjective  $\lambda$ -formulation [16] is adopted for the fuzzy nonlinear problem. For the multiobjective optimization problem, Pareto optimal sets to define its solution in this paper, the Pareto optimal solution concept is defined Eqs. (9) and (10).

$$f_i(x) \le f_i(x^*) \text{ for all } i, \qquad i = 1, \dots, k$$
(9)

$$f_i(x) < f_i(x^*)$$
 for at least one  $i$ ,  $1 \le i \le k$ . (10)

Given set of feasible solutions *P* for the problem, a solution  $x^* \in P$  is said to be Pareto optimal solution for the problem if and only if there is no any other solution  $x \in P$ , satisfying the following conditions:

## 3. Genetic algorithms approach

Genetic algorithms can be used to compute the membership functions of fuzzy optimization [18, 19]. Given some functional mapping for a system, some membership functions and their shapes are assumed for the various fuzzy variables defined for a problem [20]. The membership functions are coded as bit strings that are then concatenated. An evaluation function is used to evaluate the fitness of each set of membership functions. There are two possible ways to integrate fuzzy logic and genetic algorithms [21]. One involves the application of genetic algorithms for solving optimization and search problems related to fuzzy systems [22-24]. The other, is the use of fuzzy tools and fuzzy logic-based techniques for modeling different genetic algorithm components and adapting genetic algorithm for solving the fuzzy optimal profit problem is given below [28].

To solve the problem, given in Eq.(5), a GA is used. Since a GA seeks a set of solutions for multiple objectives in a group, it can offer several candidates to the engineer. An outline of the fuzzy multiobjective genetic algorithm for rule selection is as follows [16].

# 3.1. Representation and initialization

An initial population of size *n* is randomly generated from  $[0,1]^{k+1}$  according to the uniform distribution in the closed interval [0,1]. Let the population be

$$x_i = (\mu_{i_0}, \mu_{i_1}, \dots, \mu_{i_k}), \tag{11}$$

where i = 1, 2, ..., n and  $\mu_{ij}$  is a real number in [0,1], j = 0, 1, 2, ..., k. Each individual  $x_i$  in a population is a chromosome. For each chromosome  $x_i$ , i = 1, 2, ..., n, the centroid  $eval(x_i)$  is calculated as the fitness value. The chromosomes in the population cab be rated in terms of their fitness values. Let the total fitness value of the population be  $F = \sum_{i=1}^{n} eval(x_i)$ . The cumulative fitness value for each chromosome,  $S_i = \sum_{i=1}^{m} \frac{eval(x_i)}{F}$ , m = 1, 2, ..., n, is calculated [16].

#### 3.2 Calculation of the fitness value for each chromosome

This paper calculates the fitness value,  $eval(x_i)$  for each chromosome  $x_i$  (i = 1, 2, ..., n) as follows. For each chromosome  $x_i$  membership function values for the objectives and constraints are first calculated. If  $(\mu_{\min})_i$ , is the membership function value corresponding to the *i*th chromosome,  $x_i$ , then

$$(\mu_{\min})_i = \min\{ \mu_0[f_0(x_i)], \mu_1[f_1(x_i)], \dots, \mu_k[f_k(x_i)]\}, k = 1, 2, \dots, m$$
(12)

The fitness value for  $\mu_i$  is calculated as

$$eval(x_i) = (\mu_{\min})_i \tag{13}$$

#### 4. Numerical example

#### 4.1 Spring design

A helical compression spring needs to be designed for minimum volume and for minimum stress that show in Figure 1. Three variables are identified: The design variables are the number of spring coils N, the outside diameter of the spring, D and the spring wire diameter d. this example contains integer, discrete and continuous variables. Of these variables, N is an integer variable, d is a discrete variable and D is a real-parameter variable. Donating the variable vector  $x = (x_1, x_2, x_3) = (N, d, D)$ , write multiobjective fuzzy optimization problem as follows:

Minimize 
$$\begin{cases} f_1(x) = 0.25\pi^2 x_2^2 x_3(x_1 + 2) \\ f_2(x) = \frac{8KP_{\max} x_3}{\pi x_2^3} \end{cases}$$
(14)

Subject to  $g_1(x) = \ell_{\max} - \frac{P_{\max}}{k} - 1.05(x_1 + 2)x_2 \ge 0,$  (15)

$$g_{2}(x) = x_{2} - d_{\min} \ge 0, \tag{16}$$

$$g_{3}(x) = D_{\max} - (x_{2} + x_{3}) \ge 0, \tag{17}$$

$$g_{4}(x) = C - 3 \ge 0, \tag{18}$$

$$g_{5}(x) = \delta_{pm} - \delta_{p} \ge 0, \tag{19}$$

$$g_{_{6}}(x) = \frac{P_{_{\max}} - P}{k} - \delta_{_{w}} \ge 0, \tag{20}$$

Table 1: GA parameters for the spring design problem

Population	100
Generations	3000
Reproduction type	2 points crossover
Selection type	Rank selection
Mutation probability	0.005
Reproduction of Crossover	0,85

$$g_{\gamma}(x) = S - \frac{8KP_{\max}x_3}{\pi x_3^2} \ge 0,$$
 (21)

$$g_{8}(x) = V_{\text{max}} - 0.25\pi^{2}x_{2}^{2}x_{3}(x_{1}+2) \ge 0$$
(22)

The parameters used are as follows:

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615x_2}{x_3}, \ k = \frac{Gx_2^4}{8x_1x_3^3}, \ \delta_p = \frac{P}{k}, \ C = \frac{x_3}{x_2}$$

The maximum working load,  $P_{\text{max}} = 1000 lb$ , allowable maximum shear stress S = 189 ksi,  $V_{\text{max}} = 30 inch^3$ , maximum free length  $l_{\text{max}} = 14 inch$ ; minimum wire diameter  $d_{\text{min}} = 0.2 inch$ ; maximum outside diameter of the spring  $D_{\text{max}} = 3 inch$ ; preload compression force P = 300 lb; allowable maximum deflection under preload  $\delta_{pm} = 6 inch$  and deflection from preload position to maximum load position  $\delta_w = 1.25 inch$ .



Figure 1: The spring design problem

The design variables are bounded as  $x_i^{(l)} \le x_i \le x_i^{(u)}$ ; i = 1,2,3 where the limiting values are taken as  $1 \le x_i \le \frac{l_{\max}}{d_{\min}}$ ,  $3d_{\min} \le x_2 \le D_{\max}$  and  $d_{\min} \le x_3 \le \frac{D_{\max}}{3}$  [29, 30]. The GA parameters used for the multiobjective fuzzy optimization are given in Table 1.

From the results obtained by the classical optimization with the upper boundary values of the variables,  $f_1^{\text{max}} = 16.101$ ,  $f_1^{\text{min}} = 2.632$ ,  $f_2^{\text{max}} = 189000$ ,  $f_2^{\text{min}} = 72489.335$  are found.

If the above values are entered instead of Eqs. (3) and (4).

$$\mu_{f_1}(x) = \begin{cases} 0. & f_1 > 16.101 \\ \lambda_1 & 2.632 < f_1 < 16.101 \\ 1. & f_1 < 2.632 \end{cases}$$
(23)  
$$\mu_{f_2}(x) = \begin{cases} 0. & f_2 > 189000 \\ \lambda_2 & 72489.335 < f_2 < 189000 \\ 1. & f_2 < 72489.335 \end{cases}$$

In the fuzzy formulation, the objectives are normalized as

$$\mu_i(x) = \frac{f_i(x) - f_i^*}{f_i^{\max} - f_i^*}; i = 1, 2.$$
(25)

The normalization of Eq.(25) yields the minimum and maximum values of  $\mu_i(x)$  as zero and one, respectively. The equations of  $\lambda_1$  and  $\lambda_2$  membership functions will ensure us to achieve, optimum fuzzy decision by finding many  $\lambda$  parameters which ensure the equivalency. The results of  $\lambda$ -formulation denote that an overall satisfaction level of 66,60% has been achieved in the presence of multiple, conflicting, objectives and fuzzy information.

This paper applies proposed method to this problem and repeats the process for 3000 generations. At the 1135*th* generation of the chromosome and constraint design variables are obtained in the Table 2.

Membership Function	Design Variables				Min. Volume	Min. Stress
2	<i>x</i> <sub>1</sub>	$x_2$	<i>x</i> <sub>3</sub>	$x_4$	min $f_1$	min $f_2$
λ	(inch)	(inch)	(inch)	(inch)	$(inch)^3$	(inch)
0.8716	0.7753	0.5320	10.0000	0,9154	6,7534	0.0024

Table 2: The extreme solutions for the spring design problem

#### 4.2 Welded beam structure

Considered, as the two examples, a welded beam structure show in Figure 2. The depth of the weld (h), the length of the weld (l), the height (t) and thickness (b) of the beam are to be chosen so as to minimize the cost and deflection of the welded beam. There are constraints on the maximum shear stress ( $\tau$ ), bending stress in the beam ( $\sigma$ ), buckling load on the bar ( $P_c$ ) and side constraints. The multiobjective optimization problem formulated under the design variable vector  $x = [x_1, x_2, x_3, x_4]^T = [h, l, t, b]^T$  as follows:



Figure 2: The welded beam design problem

*Objective functions;* 

Minimize 
$$\begin{cases} f_1(x) = 1.1047 h^2 l + 0.04811 t b(L+l) \\ f_2(x) = \delta = \frac{4PL^3}{Et^3 h} \end{cases}$$
 (26)

Subject to 
$$g_1(x) = \tau(x) - \tau_d \le 0$$
 (27)

$$g_2(x) = \sigma(x) - \sigma_d \le 0 \tag{28}$$

$$g_{3}(x) = P_{c}(x) - P \le 0 \tag{29}$$

$$g_4(x) = h - b \le 0 \tag{30}$$

The parameters used are as follows:

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{l}{2R} + (\tau'')^2}, \ \tau' = \frac{p}{\sqrt{2hl}}, \ \tau'' = \frac{MR}{J}, \ M = P\left(L + \frac{l}{2}\right), \ R = \sqrt{\frac{l^2}{4} + \left(\frac{h+l}{2}\right)^2},$$
$$J = 2\left\{\sqrt{2}hl\left[\frac{l^2}{12} + \left(\frac{h+t}{2}\right)^2\right]\right\}, \ \sigma(x) = \frac{6PL}{t^2b}, \ P_c(x) = \frac{4.013E\sqrt{\frac{t^2b^6}{36}}}{L^2}\left(1 - \frac{t}{2L}\sqrt{\frac{E}{4G}}\right),$$

Expressions in which  $P = 6000 \ lb$ ,  $L = 14 \ inch$ ,  $E = 30 x 10^6 \ psi$ ,  $G = 12 x 10^6 \ psi$ ,  $\tau_{max} = 13600 \ psi$ ,  $\sigma_{max} = 30000 \ psi$ . The variables are initialization in the following range:

 $0.125 \le h, b \le 5.0$  and  $0.1 \le l, t \le 10.0$  [29, 30]. Constraints and objective functions are handled using  $\lambda$  formulation.  $\lambda$  parameters of 1 and 0 are used for the first and second objective functions, respectively.

The GA parameters used for the multiobjective fuzzy optimization are given in Table 2. The results of constrained minimization of the individual objective functions yield  $x_1^* = \{0.2443, 6.2152, 8.2986, 0,2443\}$   $f_1^{\min} = 2,3815 inch^3$ ,  $f_2^{\max} = 0.0157 inch$  and  $x_2^* = \{1.5574, 0.5434, 10.0000, 5.0000\}$   $f_1^{\max} = 36.4403 inch^3$ ,  $f_2^{\min} = 0.0004 inch$ . The normalization of Eq.(25) yields the minimum and maximum values of  $\mu_i(x)$  as zero and one, respectively. This paper applies proposed method to this problem and repeats the process for 3,000 generations. At the 1425<sup>th</sup> generation of the chromosome and constraint design variables are obtained. The results of applying the fuzzy multiobjective genetic algorithm presented in this paper to the welded beam design example are presented in Table 3.

Membership Function	Design Variables			Min. Volume	Min. Stress
λ	<i>x</i> <sub>1</sub>	$x_2$ (inch)	$x_3$ (inch)	λ	<i>x</i> <sub>1</sub>
0.6660	9	0.3805	1.8149	0.6660	9

 Table 3: The extreme solutions for the welded beam design problem

# **5.** Conclusions

This study has investigated the genetic algorithm approach to solving fuzzy optimization equations by using the membership functions of fuzzy parameters. In this approach discusses a multiobjective optimization problem with genetic algorithm and fuzzy  $\lambda$ -formulation equations.

Pareto optimal solutions for the problem of finding a genetic algorithm are used to derive the satisficing decisions. The feasibility of solutions is always maintained during the crossover and mutation operations. The proposed genetic algorithm is capable of determining the satisficing solutions set for both linear and nonlinear problems.

The results of this study may lead to the development of effective genetic algorithms for solving general fuzzy optimization problems. In summary, the fuzzy concept of the proposed genetic algorithm approach is different and gives almost the same results as the traditional methods. The results are encouraging and suggest immediate application of the proposed method to other more complex engineering design problems (Refer to Tables 2 and 3 results).

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