Modeling of the Mechanical Response of Cantilever Beams

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Abstract

The mechanical properties of cantilevers used in atomic force microscopy (AFM) play a key role in determining their ability to perform specific tasks. In particular, the bending of the cantilevers, their resonance frequencies and characteristic functions have to be optimized for samples with a given linear spring constant and for different scanning speeds. This work treats uniform cantilevers having a rectangular section and presents their mechanical properties both algebraically and numerically using Finite Element Method with the software ANSYS.

Key words: cantilever, beam, force, bending, stress

1. Introduction

In the last decade, several research groups observed that microcantilevers can transduce a number of different signal domains, e.g. mass, temperature, heat, electromagnetic field, stress, into a mechanical deformation: either a bending or a change in the resonance frequency.

The cantilever beam is an extremely useful model such as a force sensor in atomic force microscopy.

In this paper, we propose to model and simulate a cantilever with rectangular section. We will finish this work by a simulation using the Finite Element Method which will enable us to obtain the deformation and the stress repartition according to the force applied at the free end of the cantilever. Figure 1 shows an image of five rectangular cantilevers with different lengths, designed for stress measurements.

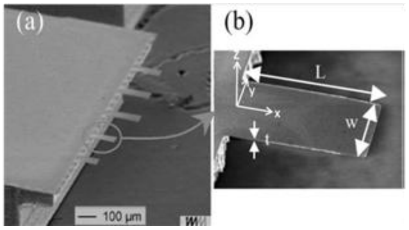

Figure 1. (a) SEM image of five rectangular cantilevers of different lengths. (b) Zoomed image of a cantilever with length L = 100 µm, Width w = 40 µm and thickness t = 0.5 µm[1]

2. Cantilever beam flex under force applied

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Under a specific force F_z applied (figure 2) at the free end of the cantilever, this last becomes deformed in each point.

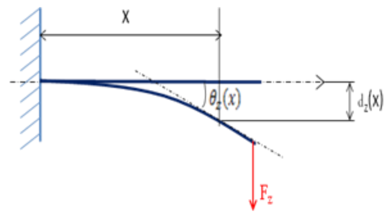

Figure 2. 1D representation of the cantilever Flex under F_z

The equation of bending of the cantilever is given by [2]:

$$\frac{\partial^2}{\partial x^2}\delta_z(x) = \frac{M_z(x)}{\hat{E}.I_z} \qquad \text{Eq. (1)}$$

Where:

- The bending moment $M_z(x)$ (expressed in N.m) is given by:

$$M_z(x) = (L - x).F_z \qquad \text{Eq. (2)}$$

- The moment of inertia I_z (expressed with m^4) is given by:

$$I_z = \frac{w.t^3}{12} \qquad \text{Eq. (3)}$$

- The effective Young's modulus Ê, is given by [3]:

$$\hat{E} = E/(1-\nu^2) \qquad \text{Eq. (4)}$$

E is the Young's modulus and υ the Poisson's ratio of material

2.1. Displacement

The clamping of the cantilever imposes the following boundary conditions:

$$\begin{cases} \delta_z(x=0) = 0 \\ \delta_z^{'}(x=0) = 0 \end{cases}$$

$$\delta_z''(x=L)=0\left[car M(x=L)=0\right]$$

By solving Eq. (1), we can get the flex $\delta_z(x)$ at any point of the cantilever, which is given by:

$$\delta_z(x)=\frac{x^2.(3.L-x)}{6.\hat{E}.I_z}.F_z \qquad \text{Eq. (5)}$$

At the free end of the cantilever, where x = L, the displacement is:

$$\delta_{z\max}=\delta(x=L)=\frac{L^3}{3.\hat{E}.I_z}.F_z=\frac{4.L^3}{\hat{E}.w.t^3}.F_z \qquad \text{Eq. (6)}$$

2.2. Linear Spring Constant

The cantilever linear spring constant, K_z, is defined as the ratio of the force applied at the free end to the resultant displacement at x = L:

$$K_z=\left|\frac{F_z}{\delta_z(L)}\right|=\frac{\hat{E}.w.t^3}{4.L^3} \qquad \text{Eq. (7)}$$

2.3. Slope

The slope of the cantilever at a point x $\theta_z(x)$ is given by:

$$\theta_z(x)=\frac{\partial}{\partial x}\delta_z(x)=\frac{(2L-x)x}{2\hat{E}I_z}.F_z \qquad \text{Eq. (8)}$$

Which, at the free end of the cantilever, where x = L, gives:

$$\theta_{z\max}(L)=\frac{L^2}{2.\hat{E}.I_z}.F_z \qquad \text{Eq. (9)}$$

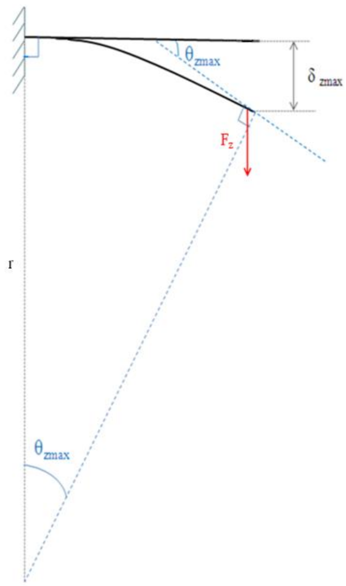

Figure 3. Schematic of the maximum displacement, slope and the radius of curvature

2.4. Cantilever Beam Stress Distribution

The stress equation at any point of the cantilever is given by [4]:

$$\sigma_l(x,y) = \frac{M_z(x)}{I_z}.z \qquad \text{Eq. (10)}$$

Where z is the distance from the neutral axis. Since the maximum stress will occur at the upper and lower surfaces at the fixed end; $z_{max} = \pm\ t\ /\ 2$. Therefore, the maximum stress can be expressed as:

$$\sigma_{\max}(x,z) = \frac{M_{z(x=0)}}{I_z}.(\pm\frac{t}{2})$$
$$\sigma_{\max}(x,z) = \pm(\frac{F_z.L}{I_z}.\frac{t}{2}) \qquad \text{Eq. (11)}$$

Inserting I_z and F_z using Eq. (3) and Eq. (6), the stress equation reduces to:

$$\sigma_{\max} = \pm\frac{3.\hat{E}.t}{2.L^2}.\delta_{z\max} \qquad \text{Eq. (12)}$$

This equation and equation 6 shows that the maximum deflection is most influenced by thickness and length, while the stress is most influenced by length.

3. Simulation using ANSYS

Then, in order to check the conformity of the algebraically method and validate the results obtained, we propose to solve the same model with another method, the finite element method. The obtained results are given in Figure 4, 5 and 6.

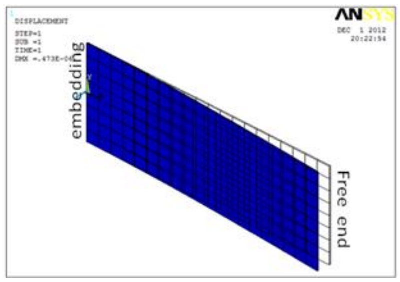

Figure 4. Deformed and undeformed shape of the beam

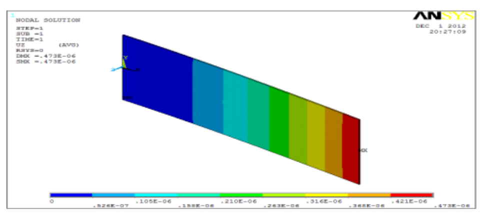

Figure 5. Flex of the beam

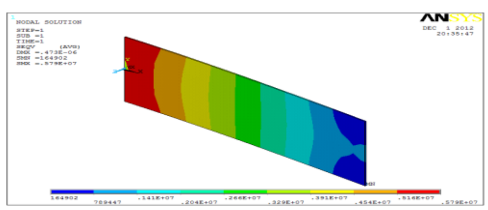

Figure 6. Repartition of the stress

Table 1. Comparison of theoretical and by simulation values of the linear spring constant

	K_z (N/m)
Theoretically	0.212
By simulation	0.213

Conclusions

In this work we have study the mechanical response of the cantilever. By modeling the mechanical behavior of cantilever theoretically and by simulation with the software ANSYS, we based on the linear spring constant and the stress repartition. The study of the stress repartition shows that the maximum is located at a rigid part of a cantilever.

In conclusion, this paper has demonstrated that it is possible to model, with good precision, the mechanical behavior of the cantilever.

The resolution of this model gives results which are in good accuracy with those obtained by numerical method.

Acknowledgements

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