Numerical Study of the Earth Pressure Distribution on Cylindrical Shafts

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Abstract:

Vertical shafts are widely used as temporary or permanent earth retaining structures for different engineering applications (e.g. tunnels, pumping stations and hydroelectric projects). Determining the earth pressure acting on the shaft lining system is essential to a successful design. Several theoretical methods have been proposed for the calculation of the active earth pressure on cylindrical retaining walls supporting granular . However, the earth pressure distribution obtained using these methods was found to vary significantly and depend on the adopted method of analysis. In addition, the required wall movement to reach the calculated pressures is yet to be understood. In this paper, numerical computations using *FLAC* are reported to evaluate the evolution of the distribution of earth pressure on a cylindrical wall embedded in granular material and subjected to radial displacement. The numerical results are discussed and compared with recent experimental results and theoretical solutions

Key words: Axisymmetric earth pressure, numerical modellig, soil, interaction, plasticity.

1. Introduction

Circular excavations are often carried in the construction of underground storage tanks, hydraulic and power facilities, manholes, inspection or access chambers and service entrances. As such, circular vertical shafts are often employed as the retaining systems for these excavations and adopted as the starting and ending sections for underground tunnelling and pipe jacking projects. Several attempts have been made to extend plane strain active earth pressure methods to study the active earth pressure against cylindrical shafts in cohesionless media. Westergaard [1] and Terzaghi [2], proposed analytical solutions; Prater [3] used the limit equilibrium method; and Berezantzev [4], Cheng and Hu [5], Cheng et al. [6], Liu and Wang [7], Liu et al. [8] used the slip line method. In contrast to the classical earth pressure theories, where the active earth pressure calculated using the Coulomb or Rankine method are essentially the same, the distributions obtained for axisymmetric conditions may differ considerably depending on the chosen method of analysis, as discussed below. In addition, the required wall movement to reach the calculated pressures is yet to be understood. The objective of this study is to investigate the active earth pressure on cylindrical shaft linings installed in cohesionless ground and the required displacement for establishing active conditions by numerical approach using the explicit finite

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difference code FLAC [9] (Fast Lagrangian Analyses of Continua). The results are compared to published experimental results.

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2. Theoretical methods

The earliest effort to investigate the state of stress around a cylindrical opening in soil was made by [1], who studied the stress conditions around small unlined drilled holes, based on the equilibrium of a slipping soil wedge. Terzaghi [2] extended Westergaard's theory to large lined holes, thus proposed a method to calculate the minimum earth pressure exerted by cohesionless soil on vertical shafts liners. He determined the equilibrium of the sliding soil mass assuming $\sigma_\theta = \sigma_v = \sigma_1$ and $\sigma_r = \sigma_3$ inside the plastic zone and employing the Mohr–Coulomb yield criterion. Terzaghi proposed the use of a reduced angle of internal friction of the sand, $\phi^* = \phi - 5$ for $30° < \varnothing < 40°$, to account for the effect of the nonzero shear stresses in the solution.

Berezantzev [4] extended the slip line method to calculate the earth pressure acting on cylindrical walls. Under active conditions Berezantzev assumed that inside the plastic zone the tangential and radial stresses are equal to the major and minor principal stresses, respectively ($\sigma_\theta = \sigma_v = \sigma_1$ and $\sigma_r = \sigma_3$). Thus λ, which is defined by the stress ratio σ_θ / σ_v is equal 1. The governing equations took the form of two partial differential equations that he solved using the Sokolovski step-by-step computation method.

Prater [3] using limit equilibrium adapted Coulomb wedge theory for axi-symmetric conditions assuming a conical failure surface. Prater argued that λ is a decisive parameter whose value should range between K_a and K_o and not equal to unity as was implicitly assumed by [2,4].

Cheng and Hu [5] extended Berezantzev's theory to develop a more general solution considering a variable earth pressure coefficient, λ. They found that the case of $\lambda = 1$ produces the lowest lateral pressure. The upper and lower bounds of the lateral earth pressure can then be obtained using $\lambda = Ko$ and $\lambda = 1$, respectively. For $\lambda = 1$ the earth pressure is the same as that calculated using the Berezantzev method.

3. Experimental investigations

Several studies have been conducted to measure the earth pressure distribution due to the installation of a model shaft in granular material. Fujii et al. [10] conducted centrifuge tests to study the effects of wall friction and soil displacements on the earth pressure distribution around rigid shafts. The experimental results for dense sand ($\phi = 42°$, $\gamma = 14.7$ kN/m^3) (Fig. 1) show good agreement with the theoretical solution [4]. Little change in the measured earth pressure was reported at displacements greater than 1% of the wall height, H (6.6% of the shaft radius), and the wall friction was found to have a negligible effect on the measured earth pressure distribution.

Imamura et al. [11] Imamura developed a model shaft similar to that used by [10]. They concluded that the earth pressure decreases with increasing wall displacement until it coincides with Berezantzev's solution at a wall displacement that corresponds to 0.2% of the wall height, H (1.6% of the shaft radius).

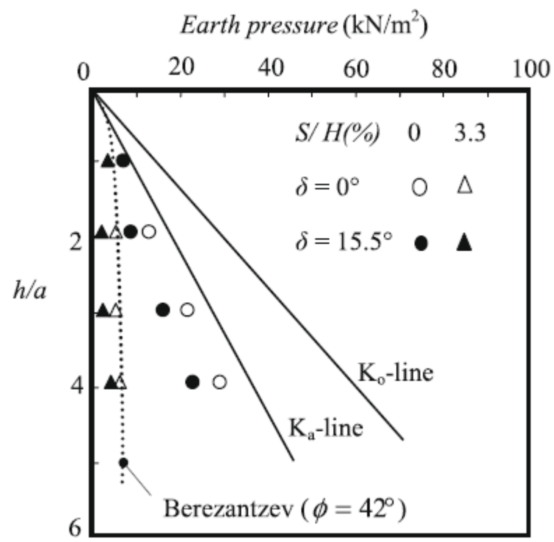


Figure 1. Semi-cylindrical model shaft and earth pressure distribution for smooth and rough walls [10]

Chun and Shin [12] performed model tests to study the effects of wall displacement and shaft size on the earth pressure distribution using a mechanically adjustable semi-circular shaft. The experimental results indicate that earth pressure decreased with increasing wall movement and became minimum when the wall movement reached 1.87 mm corresponding to 0.25% of the wall height. In figure 2, the earth pressure calculated from [2,4] methods are shown for comparison. It appears from this comparison that the measured earth pressure decreases with increasing wall displacement until it coincides between Berezantzev and Terzaghi's solutions at a wall displacement that corresponds to 0.25% of the wall height, H.

Tobar and Meguid [13] conducted a series of tests under normal gravity to investigate the changes in lateral earth pressure due to radial displacement of the shaft lining. The developed apparatus allowed for the modelling of both the full geometry of the shaft and the radial displacement of the lining. For coarse dry sand (ϕ = 41°; γ= 14.7 kN/m^3), the experimental results showed that the axisymmetric active earth pressure fully developed when the wall displacements, S, ranged between 0.2% and 0.3% of the wall height, H. They concluded that for S≥.1% H, the measured pressures (figure 3) fell into the range predicted by Cheng and Hu (2005); and that at S≥.3% H, the measured pressures closely followed the pressure distributions calculated using Terzaghi (1943) [2] and Berezantzev[4] (1958) methods.

By comparing the experimental results with the analytical solution of [3] Prater (1977), which is based on Coulomb's wedge analysis under axisymmetric conditions and a value of $\lambda=K_o$, it can be seen that the solution computes a zero value of earth pressure at a normalized depth h /a of about 9, which is considered inconsistent with experimental data.

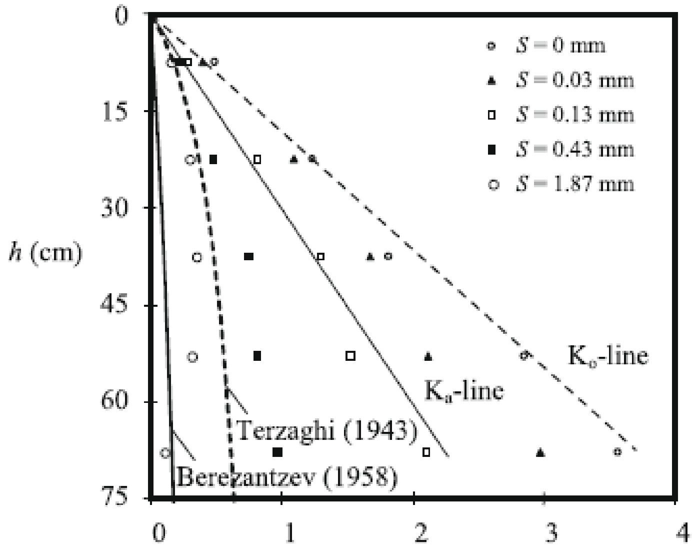


Figure 2. Semi-cylindrical model shaft and the measured earth pressure using a shape aspect ratio, H/a, of 4.286 [12]

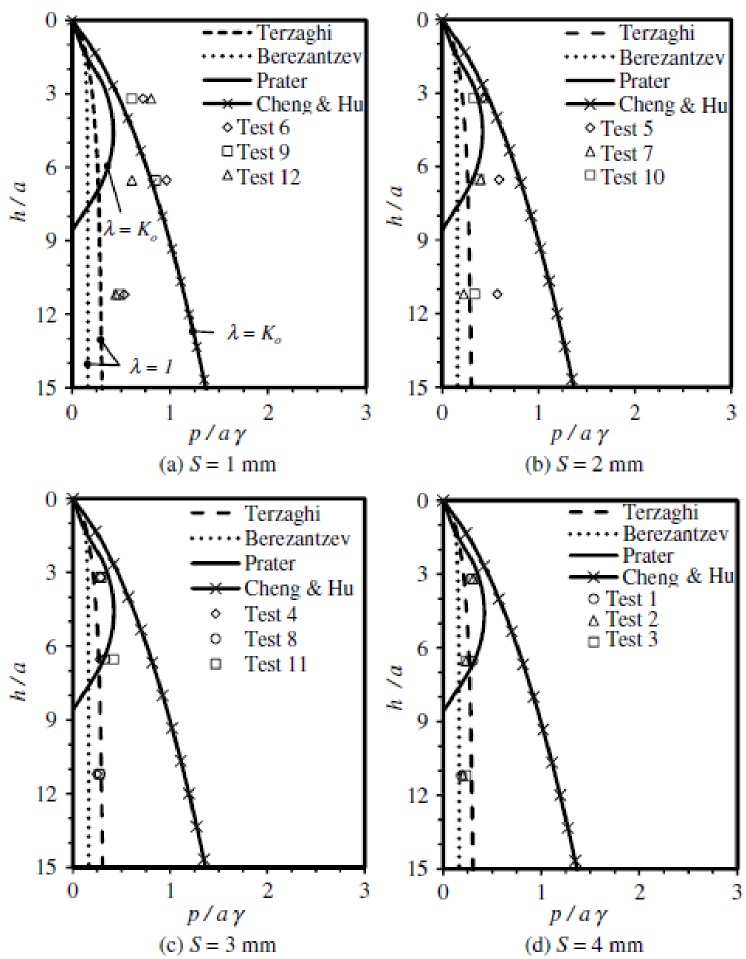


Figure 3. Comparison of measured and theoretical earth pressures along the shaft at (a) 1-mm; (b) 2-mm; (c) 3-mm; and (d) 4-mm wall movement [13]

Using different values of the coefficient λ, Cheng and Hu (2005) [5] proposed bounds for the earth pressure distribution based on slip line analysis. The upper bound is derived using $\lambda = K_o$, whereas the lower bound is derived using λ=1, which reduces the solution to the one proposed by [4] Berezantzev (1958).

Based on the above studies it can be concluded that, for axisymmetric excavations under active conditions, the theoretical and the experimental results showed that the axisymmetric active earth pressure distribution for a cylindrical walls does not increase linearly with depth as it does in long vertical walls under plane strain conditions. As the soil movement increases, the normalized pressure distribution reduces until a constant value (independent of the depth) is reached at the ultimate state. However theoretical solutions show high discrepancy related to the hypothesis concerning the lateral stress coefficient which cannot be determined from the theories. From the theoretical and experimental overviews, numerical analysis are carried out with no hypothesis on the lateral stress coefficient and the results are compared with theoretical and the experimental results.

4. Numerical modelling

The experimental study of the earth pressure distribution on cylindrical shafts reported by Tobar and Meguid (2011) [13] are numerically investigated by using the computer code FLAC which is a commercially available finite difference explicit program.

The soil behaviour is modelled by the elastic-perfectly plastic Mohr–Coulomb model encoded in FLAC code. All subsequent results are given for γ=14.7kN/m^3, elastic bulk modulus K = 30 MPa and shear modulus G = 11.25 MPa , internal friction angle ϕ=41° , cohesion c=0.

The proposed modeling procedure of the active earth pressure distribution on cylindrical shafts follows two steps:

- In the first one, the shaft installation and the geostatic stresses are computed assuming fixed shaft connected to the soil via interface element. At this stage the strength of the interface elements are assigned to be null and some stepping is required to bring the model to equilibrium;
- In the second step, a radial velocity of 10^{-6} m/step towars the shaft axis was applied to the gridpoints representing the wall shaft until a steady plastic flow is achieved (i.e. until a constant pressure on the shaft wall is reached). As the level of errors in such calculation scheme by FLAC depends on the applied velocity, a low velocity is recommended.

The mesh size is fine near the wall where deformations are concentrated. As a general rule for the boundary conditions, the bottom boundary is assumed to be fixed in the vertical direction, the right and left lateral boundaries are fixed in the horizontal directions. For axi-symmetry problem, structural elements incorporated in FLAC don’t work. Therefore, the shaft wall is modelled by thin fixed membrane elements connected to the soil grid via interface elements attached on both sides. Fig. 4 shows the axi-symmetric mesh retained for this analysis and plastic zone corresponding to limit state.

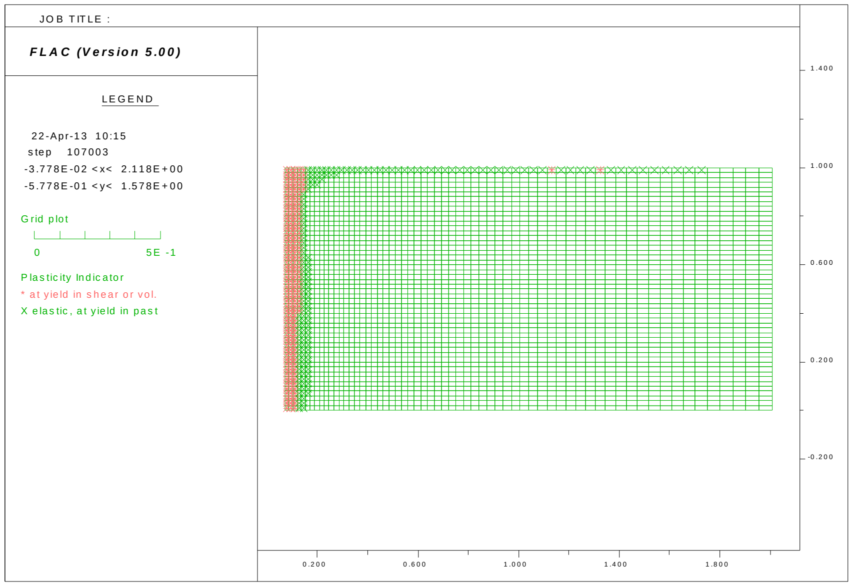


Figure 4. Mesh used and plastic zone

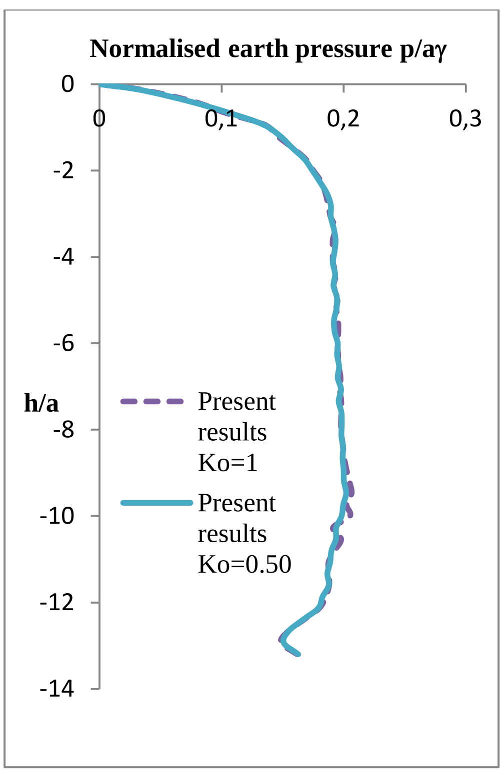


Figure 5. Active earth pressure distribution

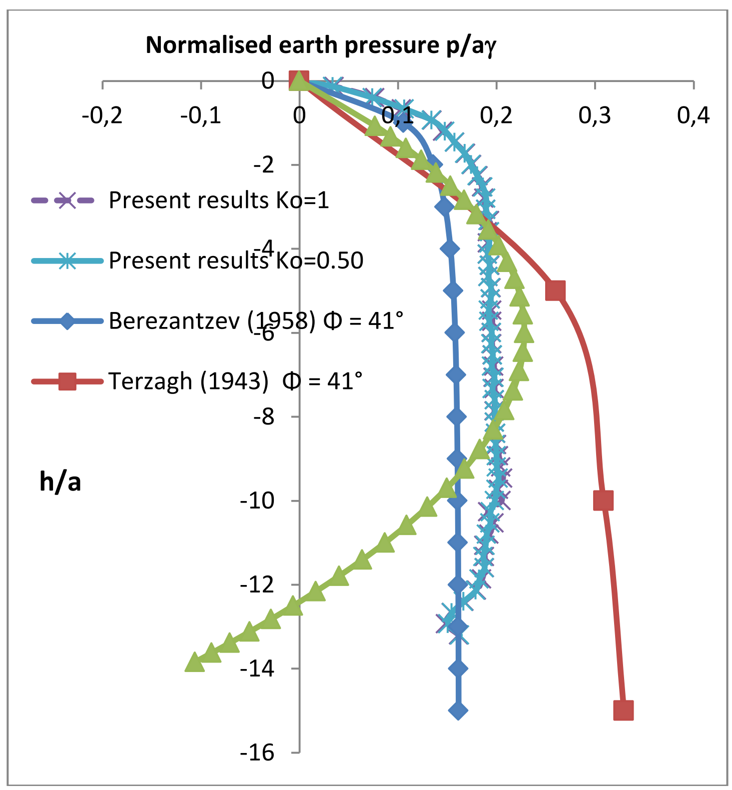
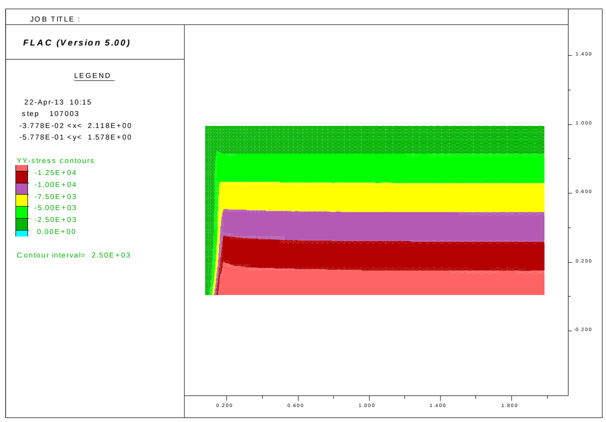


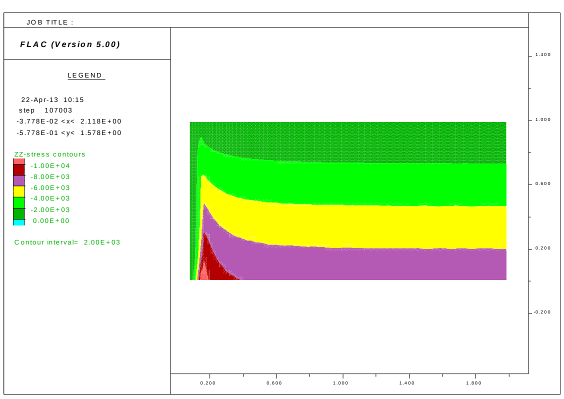
Figure 6. Comparison of active earth pressure distribution

Fig. 5 shows the numerical results of the active earth pressure distribution with shaft depth for two values of the earth pressure at rest Ko=0.5 and Ko=1. The results confirm that variation in practical range of the earth pressure coefficient at rest K_0 do not have any significant influence on the axi-symmetric active earth pressure distribution.
Fig. 6 shows the comparison the present results to theoretical solutions [2,3,4] and experimental results [13], it can be noted that the present numerical results agree well with the measured earth pressure [13] and the solutions [2,4], both assuming a value of λ equal to unity, provided that enough soil movement is allowed. As shown in Fig. 6, Prater's method predicts a different pressure distribution with a zero earth pressure at some depth below the surface.

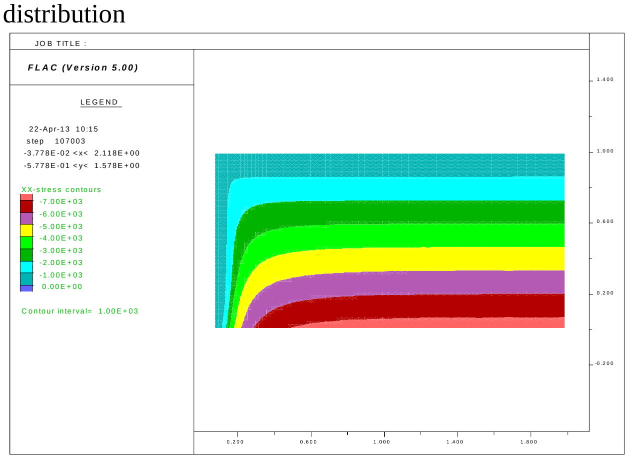
The radial, tangential and vertical stress distributions in the medium and in radial direction at the half of the model height are plotted in Figs. 7 and 8 respectively. The results show a slight increase of the vertical stress (σ_v) in the elastic region near the elastic-plastic interface followed by a drastic reduction in the plastic region. This behaviour indicates that arching in vertical planes is formed. The tangential stresses (σ_θ) increase toward the shaft wall in the elastic region, followed by a brutal decrease and converge to vertical stresses in the plastic region. Also, the radial stresses (σ_r) decrease toward the shaft wall accentuated in the plastic zone.



-a- σ_v distribution



-b- σ_θ distribution



-c- σ_r distribution

Figure 7. Stresses distribution at limit state

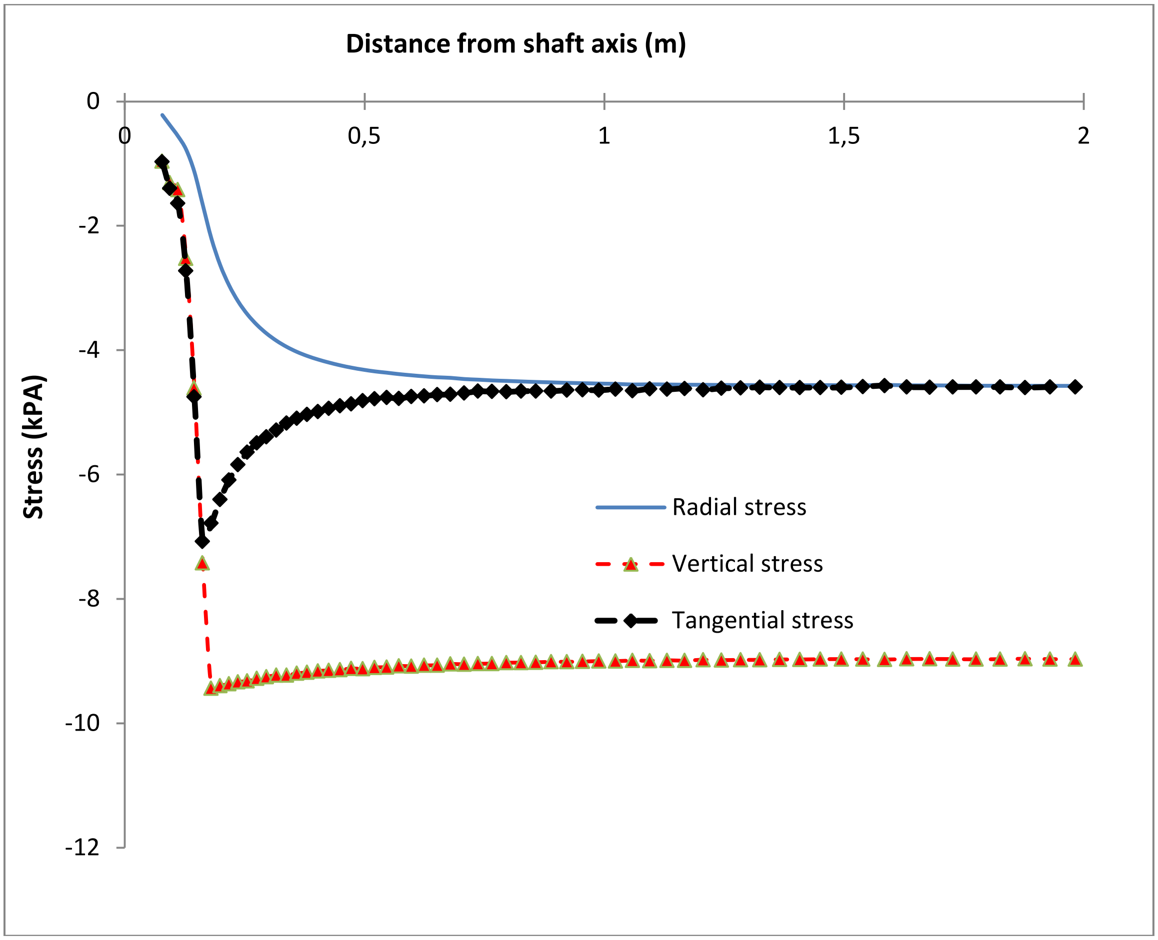


Figure 8. Stresses distribution at 0.4h from the base

Conclusions

A numerical study was performed for a physical model to investigate the earth pressure distribution on a cylindrical shaft. The results were compared with both some theoretical solutions and experimental measurements of the physical model. The numerical, the theoretical and the experimental results show that the axi-symmetric active earth pressure distribution for cylindrical shafts does not increase linearly with depth as it does plane strain conditions. The wall movement induces a reduction of the earth pressure distribution until a constant value at the ultimate state for high friction soil. The theoretical solutions show high discrepancy related to the hypothesis concerning the lateral stress coefficient $\lambda=\sigma_\theta/\sigma_v$ which cannot be determined from the theories. A good agreement was noted between the present numerical modelling results, experimental results [13] and theoretical solutions of [2,4] which both assuming a value of λ equal to unity. FLAC numerical results show a drastic reduction of the vertical stress (σ_v) and tangential stresse (σ_θ) in the plastic region against the shaft wall and confirm the hypothesis $\lambda=\sigma_\theta/\sigma_v=1$ assumed by [2,4].

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