Fuzzy Linear Programming Approach for Deciding the Production Amount of Different Fruit Juice Types

*[1]Çağatay Teke, [2]Tuğba Yüksel, [1]Özer Uygun
*[1]Faculty of Engineering, Department of Industrial Engineering Sakarya University, Turkey
[2]Master Student, Faculty of Engineering, Department of Industrial Engineering Sakarya University, Turkey

Abstract

In recent years, rapid and correct decision making is crucial for both people and enterprises. However, uncertainty makes decision-making difficult. Fuzzy logic is used for coping with this situation. Thus, fuzzy linear programming models are developed in order to handle uncertainty in objective function and the constraints. In this study a problem in a fruit juice production factory is investigated, required data is obtained and the problem is figured out as a fuzzy linear programming model. The model is solved using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of production for each fruit juice type in order to gain maximum profit.

Key words: Fuzzy linear programming, fuzzy logic, linear programming, decision making, fruit juice

1. Introduction

Decision Making (DM) in a productive environment consists of criteria and methodologies, which help managers in the process of selecting the best choice among a set of alternatives. DM often relies on Decision Support Systems (DSS) tools adopting Linear Programming (LP) theories [1].

Fuzzy set theory appears to be an ideal approach to deal with decision problems that are formulated as linear programming models but with imprecision parameters. [2] Indeed, fuzzy modeling allows working with variables that well represent the concept of uncertainty when available historical data are not sufficient to define probability distributions (i.e. innovative manufacturing systems). In fuzzy LP, different techniques are available. The approach based on ambiguous coefficients of the objective function is called possibilistic programming. A common feature in the above approaches is the preliminary defuzzification of the fuzzy variables. The resulting problem is solved by classic optimization, obtaining a single final solution. Several LP problems can be obtained from an original fuzzy LP problem by using different criteria such as the probability maximization to reach a minimum specified profit [1].

In fuzzy decision making problems, the concept of maximizing decision was introduced by Bellman and Zadeh. This concept was adapted to problems of mathematical programming by Tanaka et. al. Zimmermann presented a fuzzy approach to multi-objective linear programming problems in his classical paper. Lai and Hwang, Tong Shaocheng, Buckley, among others, considered the situation where all parameters are in fuzzy. Lai and Hwang assume that the parameters have a triangular possibility distribution. They use an auxiliary model which is solved

*Corresponding author: Address: Faculty of Engineering, Department of Industrial Engineering Sakarya University, 54187, Sakarya TURKEY. E-mail address: cteke@sakarya.edu.tr, Phone: +902642957404

by multi-objective linear programming methods. Nagi and Buckly obtain an optimal solution by using possibility concepts. Tong Shaocheng defines a goal for the objective function and they by using Zedeh's 'Min' operator obtains an optimal solution. In the recent years, numerous methods for comparison of fuzzy numbers have been suggested in the literature. Maleki, Tata and Mashinchi used Rouben's method for comparison of fuzzy numbers and obtain optimal solution. Jing-Shing Yao, Kweimei Wu used ranking fuzzy numbers based on decomposition principle for comparison of fuzzy number and obtain on optimal solution [3].

2. Fuzzy Sets

2.1. Fuzzy Sets Theory

Fuzzy set theory derives from the logic underlying the modes of reasoning, which are approximate rather than exact. The importance of fuzzy logic derives from the fact that the modes of human reasoning, and especially, common sense reasoning is approximate in nature; both do not well-define boundaries of referred objects, such as young man, high temperature, big size, and so on. Furthermore, everything is a matter of degree; such as not quite young, high to some extent, and so on [6].

Fuzzy set theory is a generalization of traditional crisp set theory. As the underlying formulation of any optimization problem relies on the set structure, optimization problems under an uncertain environment can be reformulated using fuzzy sets [4].

The classical set theory is built on the fundamental concept of "set" of which an individual is either a member or not a member. A sharp, crisp, and unambiguous distinction exists between a member and a nonmember for any well-defined "set" of entities in this theory, and there is a very precise and clear boundary to indicate if an entity belongs to the set. In other words, when one asks the question "Is this entity a member of that set?" The answer is either "yes" or "no." This is true for both the deterministic and the stochastic cases. In probability and statistics, one may ask a question like "What is the probability of this entity being a member of that set?" In this case, although an answer could be like "The probability for this entity to be a member of that set is 90%," the final outcome (i.e., conclusion) is still either "it is" or "it is not" a member of the set. The chance for one to make a correct prediction as "it is a member of the set" is 90%, which does not mean that it has 90% membership in the set and in the meantime it possesses 10% non-membership. Namely, in the classical set theory, it is not allowed that an element is in a set and not in the set at the same time. Thus, many real-world application problems cannot be described and handled by the classical set theory, including all those involving elements with only partial membership of a set. On the contrary, fuzzy set theory accepts partial memberships, and, therefore, in a sense generalizes the classical set theory to some extent [5].

3. Fuzzy Linear Programming

3.1. Linear Programming

A Linear Programming (LP) problem is a special case of Mathematical Programming problem. From an analytical perspective, a mathematical program attempts to identify an extreme (minimum or maximum) point of a function, which furthermore satisfies a set of constraints. Linear programming is the objective function and the problem constraints are linear [6].

A classical model of LP, also called a crisp LP model, may have the following formulation:

$$\begin{aligned} &\text{Max} \quad Cx \\ &\text{s.t.} \quad A_i x \leq b_i, \qquad i=1, \dots, m, \end{aligned}$$

in which x is an $n \times 1$ alternative set, C is a $1 \times$ n coefficients of an objective function, A_i is an $m \times n$ matrix of coefficients of constraints and b_i is an $m \times 1$ right-hand sides.

The traditional problems of LP are solved with LINDO optimization software and obtain the optimal solution in a precise way. If coefficients of constraints, objective function or the right-hand sides are imprecise, in other words, being fuzzy numbers, traditional algorithms of LP are unsuitable to solve the fuzzy problem and to obtain the optimization.

In the real world, the coefficients are typically imprecise numbers because of insufficient information, for instance, technological coefficients. Many researchers formed Fuzzy Linear Programming of various types, invented approaches to convert them into crisp LP, and finally solved the problems with available software [7].

3.2. Fuzzy Linear Programming

Fuzzy linear programming (FLP) follows from the fact that classical linear programming is often insufficient in practical situations. In reality, certain coefficients that appear in classical LP problems may not be well-defined, either because their values depend on other parameters or because they cannot be precisely assessed and only qualitative estimates of these coefficients are available. FLP is an extension of classical linear programming and deals with imprecise coefficients by using fuzzy variables [8].

Now we consider the FLP Problem

$$\begin{aligned} &\text{Max } \tilde{Z} = \tilde{C}^{\mathrm{T}} x \\ &\text{s.t.} \quad \tilde{A}x \lesssim \tilde{b} \\ &\qquad\quad x \geq 0. \end{aligned}$$

The solution of this problem is to find the possibility distribution of the optional objective function Z. Many researchers had handled this problem by converting the fuzzy objective function and the fuzzy constraints into crisp ones [9].

Fuzzy linear programming model divide into parts in terms of fuzzy coefficients. For instance, while objective function is fuzzy, constraints cannot be fuzzy.

3.2.1. Objective Function is Fuzzy

In a real life, there are many situations that parameters of objective function (profit and cost) are imprecise. FLP model of this was propounded by Verdegay.

3.2.2. Right-Hand Sides are Fuzzy

There are two approach for this type of problem. While first approach concerning asimetric models belongs to Verdegay, second approach concerning simetric model belongs to Werners.

3.2.3. Right-Hand Sides and Coefficients of Constrains are Fuzzy

Negoita and Sularia developed an approach for this type of FLP model.

3.2.4. Objective Function and Constrains are Fuzzy

As it is understood the title, in this model, both objective function and constrains involve fuzziness. Zimmermann and Chanas have different approaches about it.

3.2.5. All Coefficients are Fuzzy

Sometimes, all coefficients can be fuzzy in the problem. Carlsson ve Korhonen developed the approach for this.

3.3. Zimmermann Method

A LP with a fuzzy objective function and fuzzy inequalities shown by Zimmermann is indicated as follows: [7]

$$c^T x \gtrapprox b_0$$

$$(Ax)_i \lessapprox b_i \qquad i=1,2,\ldots,m$$

$x \geq 0$

Inequality is a symmetrical model of which the objective function becomes one constraint. To write a general formulation, inequality is converted to a matrix form as [7]:

$-c^T x \lesssim -b_0$

in which

$B = \begin{bmatrix} -C \\ Ai \end{bmatrix}$ ve $b = \begin{bmatrix} -b0 \\ bi \end{bmatrix}$

The inequalities of constraint signify "be as small as possible or equal" that can be allowed to violate the right-hand side b by extending some value. The degree of violation is represented by membership function as [7]:

$$\mu_0(x) = \begin{Bmatrix} 0 & ;\ if & cx \leq b0\text{-}d0 \\ 1\text{-}\frac{b0\text{-}cx}{d0} & ;\ if & b0\text{-}d0 \leq cx \leq b0 \\ 1 & ;\ if & cx \geq b0 \end{Bmatrix}$$

$$\mu_i(x) = \begin{Bmatrix} 0 & ;\ if & (Ax)i \geq bi+di \\ 1\text{-}\frac{(Ax)i\text{-}bi}{di} & ;\ if & bi \leq (Ax)i \leq bi+di \\ 1 & ;\ if & (Ax)i \leq bi \end{Bmatrix}$$

in which *d* is a matrix of admissible violation.

By introducing the auxiliary variable λ, this problem can be transformed as follows:

$\mu_0(x) \geq \lambda$

$\mu_i(x) \geq \lambda$

$\lambda \in [0,1]$

This problem can be stated as linear programming as follows:

Max λ
s.t.

$\mu_0(x) \geq \lambda$

$\mu_i(x) \geq \lambda$

$\lambda \in [0,1]$

This problem was shown with membership functions of fuzzy objective function and fuzzy constrains as follows:

$Max\ \lambda$
$s.t.$

$$1 - \frac{b_0 - c_x}{d_0} \geq \lambda$$

$$1 - \frac{(Ax)_i - b_i}{d_i} \geq \lambda \quad ; \ \forall i$$

$\lambda \in [0,1]$

$x \geq 0$

After some simplification, fuzzy linear programming model obtain as follows:

$Max\ \lambda$ $s.t.$		$Max\ \lambda$ $s.t.$
$c^T x \geq b_0 - (1-\lambda)d_0$		$c^T x - \lambda d_0 \geq b_0 - d_0$
$(Ax)_i \leq b_i + (1-\lambda)d_i \quad ; \ \forall i$	$\equiv$	$(Ax)_i + \lambda d_i \leq b_i + d_i \quad ; \ \forall i$
$\lambda \in [0,1]$ $x \geq 0$		$\lambda \in [0,1]$ $x \geq 0$

4. Application

4.1. Problem Definition

Data used for the application was obtained from a fruit juice company. The company produce different fruit juice types. Since the expected profit and the demand of the product types are uncertain the problem is built as fuzzy linear programming model in order to determine production amounts per day for each fruit juice type for maximizing the profit. Data about the production and its constraints is given in Table 1:

Table 1: Data About the Application

Variables						
	X_1	X_2	X_3	X_4	X_5	X_6
Variable name	Peach juice	Apricot juice	Apple juice	Orange juice	Mixed fruit juice	Cherry juice
Unit profits(TRY/L)	0.81	0.77	0.68	0.72	0.71	0.60
Expected demands(L)	16500	15000	5750	6250	10500	5350
Tolerances for demands(L)	1200	1000	750	700	850	350
Labour usage(min/L)	0.420	0.396	0.456	0.426	0.492	0.438
Expected profit(TRY)	45000					
Tolerance for profit(TRY)	5000					
Daily production capacity(L)	60000					
Daily labour capacity(min)	26400					

4.2. FLP Model

Problem was modeled as daily basis. The fuzzy linear programming model of the problem is given below:

$c^T x = 0.81x_1 + 0.77x_2 + 0.68x_3 + 0.72x_4 + 0.71x_5 + 0.6x_6$

$b_0 = 45000$ $\quad d_0 = 5000$
$b_1 = 16500$ $\quad d_1 = 1200$
$b_2 = 15000$ $\quad d_2 = 1000$
$b_3 = 5750$ $\quad d_3 = 750$
$b_4 = 6250$ $\quad d_4 = 700$
$b_5 = 10500$ $\quad d_5 = 850$
$b_6 = 5350$ $\quad d_6 = 350$

Max λ
st

$0.81x1 + 0.77x2 + 0.68x3 + 0.72x4 + 0.71x5 + 0.6x6 - 5000\lambda \geq 40000$

$x_1 + 1200\lambda \leq 17700$
$x_2 + 1000\lambda \leq 16000$

$x_3+750\lambda \ \leq \ 6500$
$x_4+700\lambda \ \leq \ 6950$
$x_5+850\lambda \ \leq \ 11350$
$x_6+350\lambda \ \leq \ 5700$

$x_1+x_2+x_3+x_4+x_5+x_6 \leq 60000$

$0.420x_1+0.396x_2+0.456x_3+0.426x_4+0.492x_5+0.438x_6 \leq 26400$

$\lambda \in [0,1]$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

4.3. Problem Solution

FLP model of the problem has been solved using Lindo optimization software. Results of the solution are given below:

OBJECTIVE FUNCTION VALUE

1) 0.8819100

VARIABLE	VALUE	REDUCED COST
X_1	16641.000000	-0.000162
X_2	15118.000000	-0.000154
X_3	5838.000000	0.000136
X_4	6332.000000	-0.000144
X_5	10600.000000	-0.000142
X_6	5391.000000	-0.000120
λ	0.881910	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-0.000200
3)	0.708030	0.000000
4)	0.090025	0.000000
5)	0.567519	0.000000
6)	0.663017	0.000000
7)	0.376521	0.000000
8)	0.331509	0.000000
9)	80.000000	0.000000

10)	488.034149	0.000000
11)	0.881910	0.000000
12)	0.118090	0.000000

NO. ITERATIONS= 154
BRANCHES= 25 DETERM.= 1.000E 0

As can be seen from the solution, the company should produce 16641 L of peach juice, 15118 L of apricot juice, 5838 L of apple juice, 6332 L of orange juice, 10600 L of mixed fruit juice, 5391 L of cherry juice. Total profit of the company can be calculated as follows:

(16641x0.81)+(15118x0.77)+(5838x0.68)+(6332x0.72)+(10600x0.71)+(5391x0.6)=44409.5 TRY

5. Conclusion

In this study, a problem in a fruit juice production factory was modelled by using fuzzy linear programming. Because the model has fuzziness in both objective function and constraints, it was solved using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of production for each fruit juice type in order to gain maximum profit by meeting the demands of each fruit juice types.

References

[1] Nucci F, Cavallo C, Grieco A. A fuzzy linear programming approach to mix product selection problem. IEEE 2007;1-4244-1210-2/07.
[2] Guu SM, Wu YK. Two-phase approach for solving the fuzzy linear programming problems. Fuzzy Sets and Systems 1999;107:191-195.
[3] Gani AN, Duraisamy C, Veeramani C. A note on fuzzy linear programming problem using L-R fuzzy number. International Journal of Algorithms, Computing and Mathematics 2009;3:93-106.
[4] Liang RH, Hsu YY. Fuzzy linear programming : an application to hydroelectric generation scheduling. IEE Proc.-Gener. Transm. Distrib. 1994;6:568-574.
[5] Chen G, Pham TT. Introduction to Fuzzy Sets, Fuzzy Logic and Fuzzy Control Systems. CRC Press;2001.
[6] Dervişoğlu E. Fuzzy Linear Programming : Review and Implementation. Sabanci University. İstanbul;2005.
[7] Lee CS, Wen CG. River assimilative capacity analysis via fuzzy linear programming. Fuzzy Sets and Systems 1996;79:191-201.
[8] Ren J, Sheridan TB. Optimization with fuzzy linear programming and fuzzy knowledge base. IEEE 1994; 0-7803- 1896-X/94.
[9] Gasimov RN, Yenilmez K. Solving Fuzzy Linear Programming Problems with Linear Membership Functions. Turk J Math 2002;26:375-396.