Wave Propagation Properties in $LiNbO_3$ Using the Finite Element Method

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Abstract:

The objective of this work is to study light propagation in optical waveguides of the type Ti in-diffused in $LiNbO_3$ using the finite element method. The calculation is carried out in 2 and 3 dimensions. This has allowed us to analyze the behavior of the electrical field inside and outside the guide, which has been optimized using an improved model for the determination of the mode sizes that propagate inside the waveguide. The results obtained here are compared to those recently published.

Key words: $LiNbO_3$, integrated optics, FEM, optical communication, variational method

1. Introduction

Nowadays, an increasing attention is being given to strip waveguides formed by the diffusion of titanium (Ti) in lithium niobate ($LiNbO_3$) [1]. Such waveguides provide the basis for many promising devices for both optical fiber communication systems and optical processing applications. However, in order to use the potentiality of these devices, low loss waveguides and efficient coupling are essential.

In order to improve light confinement in optical waveguides, one must know its guiding properties which are given by the electric field distribution of the guided wave in devices based on Ti in-diffused in $LiNbO_3$ substrates. The schematic diagram of optical waveguide considered in this work is shown in Figure 1.

A plane wave is assumed to be traveling along the y-axis in the waveguide strip in which the electric field is aligned along the z-axis, as shown in Figure 1. So, the scalar equation [1-3] is:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k^2n^2(x,z)-\beta^2\right]\vec{E}_z(x,z)=0 \quad (1)$$

Analytical methods based on the variational method have shown little agreement between the theory and the experimental data [1]. Therefore, it is necessary to look into numerical approaches to resolve the scalar equation, Eq. (1). Among the most convenient methods that can be used are the finite difference method (FDM) and the finite element method (FEM) [4].

2. Theory

The Finite Element Method, which is a powerful numerical tool, is widely used in resolving

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problems in finite spatial domains. It has been successfully applied in the computational problems of electric fields [4].

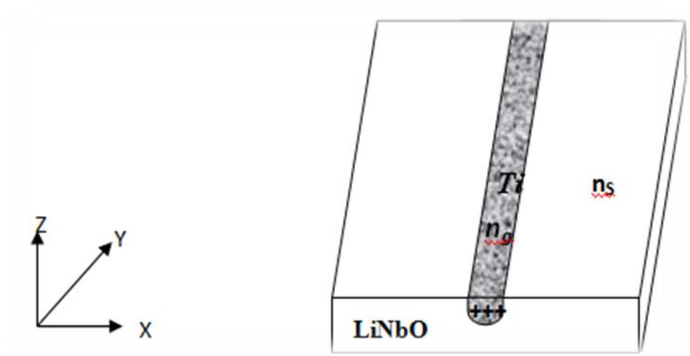

Figure 1. Schematic diagram of the in-diffused Ti:LiNbO$_3$ waveguide.
(n_s and n_g are the refractive indices of the substrate and the guide, respectively)

The idea of the method is to find an approximate solution to a differential equation after a reformulation as an integral identity called a variational form. Instead of trying to satisfy the equation nodes, we decompose the domain by subdomains called finite elements, and this requires satisfying the field equation [4].

To illustrate the principle of FEM, we take, for instance, the example of the Helmholtz equation and we try to minimize the amount of R such that:

$$\nabla^2\emptyset(x,z) + (k^2n^2(x,z) - \beta^2)\emptyset(x,z) = 0 \tag{2}$$

where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \tag{3}$$

in which n(x, z) is the refractive index profile, β is the propagation constant and k is the wavenumber.

$$R = \nabla^2\emptyset(x,z) + (k^2n^2(x,z) - \beta^2)\emptyset(x,z) \tag{4}$$

Now, we need to choose a set of functions that are linearly independent, W_n, called the projection functions, in order to balance all the integrals on each of the finite elements:

$$I_n = \int_\Omega W_n R d\Omega \tag{5}$$

Instead of solving the Helmholtz equation directly, it is replaced by its variational form:

$$I[\phi] = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2 - (k^2n^2(x,z) - \beta^2)\phi^2\right]dxdz \tag{6}$$

A set of n equations with n unknowns is obtained, and thus forming a basic system that can be written in the following matrix form:

$$[A_e]\{\emptyset_e\} = \{b_e\} \tag{7}$$

in which, the matrix $[A_e]$ is associated with the element. Its coefficients are functions of the coordinates of the element nodes. The components $\{\emptyset_e\}$ are unknown variables to the nodes of the electric field of the same element. The vector $\{b_e\}$ takes into account whatever boundary conditions present on some nodes of the considered element [4].

3. Results and discussion

In the optical waveguide based on in-diffused Ti:$LiNbO_3$, the simulation results for light wave propagation are depicted in the Figures 2 to 6. As it can be observed from the figures, the electrical field is intense inside the strip in the waveguide.

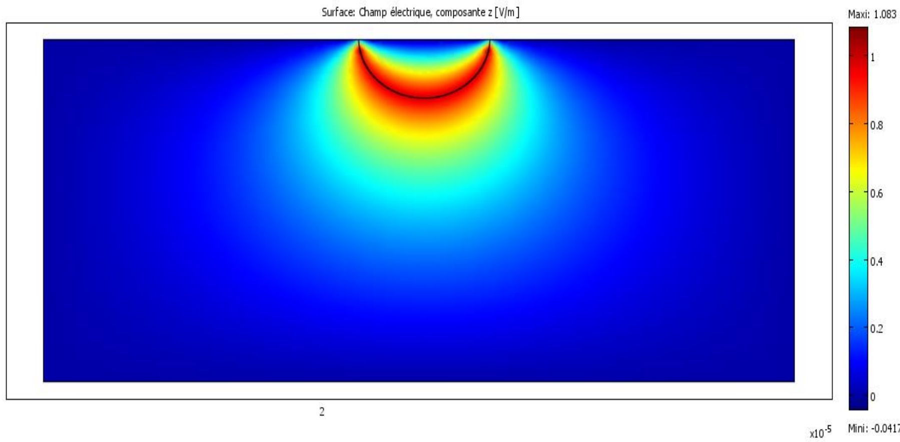

Figure 2. The electric field distribution along the waveguide.

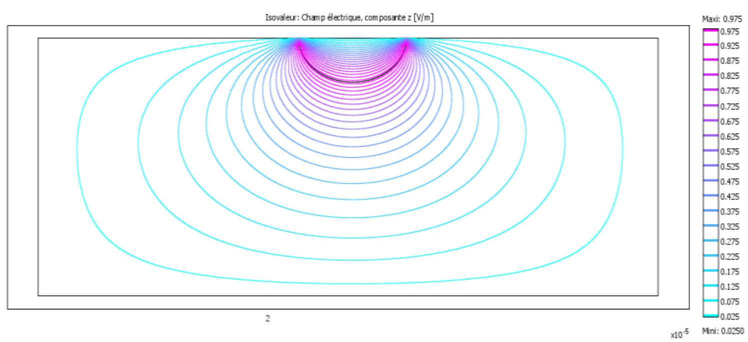

Figure 3. The Equipotential lines of the simulated electrical field distribution inside the waveguide.

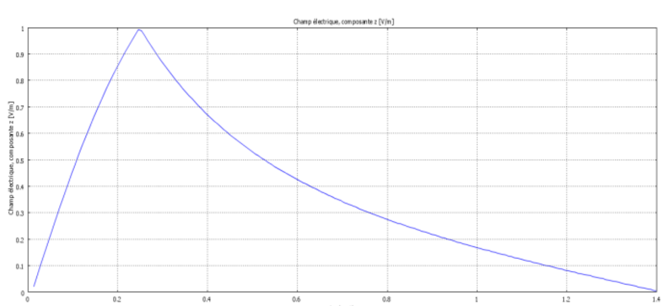

Figure 4. The electric field variation as a function of depth in the substrate.

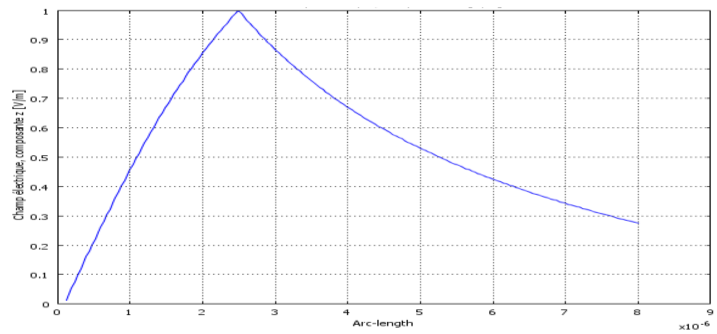

Figure 5. The electric field dependence on depth in the titanium strip.

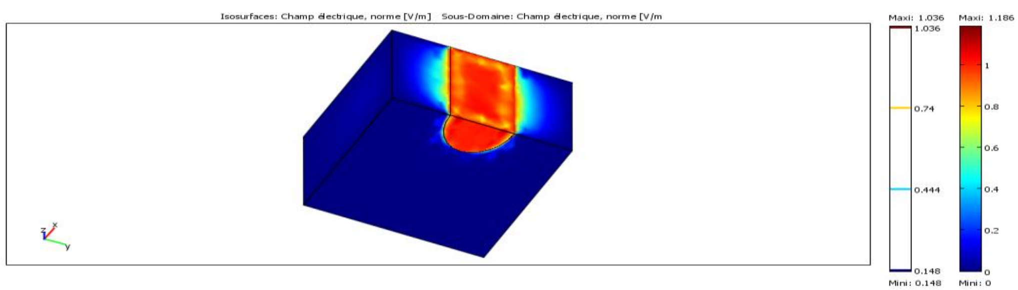

Figure 6. A 3-D plot showing the electric field distribution in an integrated optical device based on Ti:LiNbO$_3$.

We compared the results that were obtained by simulating the electrical field distribution of the type Ti:LiNbO$_3$ with those obtained by Popescu [5] and the good agreement is obtained. Figure 7 shows results by comparing those of our work with that of Popescu [5].

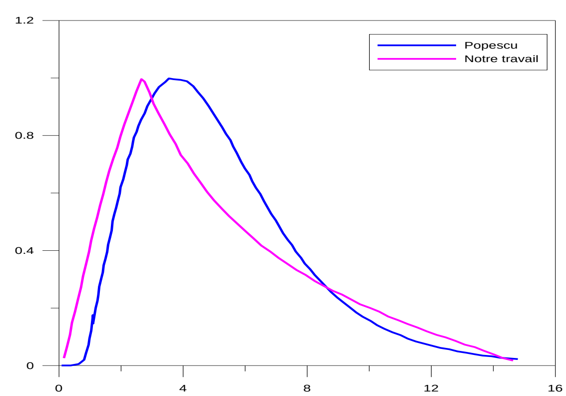

Figure 7. Comparative plot results obtained from our model and that of Popescu [5]

Conclusions

In summary, we have presented our results in simulating the properties of light propagation inside an integrated optical waveguide that is based on in-diffused Ti : $LiNbO_3$. This work allowed us to establish the fundamentals in order to analyze optical devices that are more complex in structures such as couplers, switches and modulators.

References

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