# Determination of Mechanical Properties of Elastomers Using Instrumented Indentation

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**Abstract:**

Although a great deal of work has been done in the past, most engineers find elastomers difficult to play around. They are partly right, since there is no single “modulus of elasticity” to find, but a whole function (the strain energy density) that is not known. On the other hand, the tremendous non-linearities in the deformation and the material response, combined with the incompressibility constraint, make analysis, as well as computations with FEM, a difficult and often a problematic task. Classical uniaxial tensile tests tell a very limited story about the mechanical behavior of elastomers. The situation worsens as we reduce sizes and available material volumes that need to be tested. We have realized that a modified instrumented indentation test can provide sufficient experimental information for the reconstruction of the strain energy density function. The theoretical and numerical investigations were supplemented with extensive testing of rubbers and gels.

**Key words:** elastomers, indentation, mechanical properties, micro-scales

## 1. Introduction

Although a great deal of work has been done in the past, most engineers find elastomers difficult to play around (not to mention that graduate students tend to avoid the subject!). They are partly right, since there is no single “modulus of elasticity” to fix and simplify things, but a whole function (the strain energy density) that is not known. On the other hand, the tremendous non-linearities in the deformation (large stretches and rotations) and the material response (Mooney-Rivlin, Gent, Ogden, Boyce etc.), combined with the incompressibility constraint, make analysis, as well as computations with FEM, a difficult and often dangerously problematic task. Things are bad enough regarding classical testing: uniaxial tensile tests tell a very limited story about the mechanical behavior of elastomers. The situation worsens as we reduce sizes and available material volumes that need to be tested.

Elastomers can be (almost) incompressible or compressible. We have developed indentation techniques that can assess the strain energy density to any desired level of detail. We can separate the time effects (viscoelasticity) from the purely elastic (but very non-linear) effects. Because we solved completely the problem (it rarely happens with these materials!), we have realized that a simple instrumented indentation test (one that monitors simultaneously the applied force on the indenter and the resulting vertical displacement, as functions of time) cannot provide sufficient experimental information for the reconstruction of the strain energy density function (inverse

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problem). So, here comes our second novelty: indentation under controlled prestrained conditions. The material is indented, after being subjected to various (known) levels of prestrain. The theoretical and numerical investigation was supplemented with extensive testing of elastomers (rubbers) that were independently characterized by classic tests. We created several prototype indentation tips of spherical and cone shapes and performed instrumented micro-indentation tests (depth of order mm). Very recently, we have initiated a collaboration with HISITRON, Minesota, where our methods will be tried with instrumented nano-indentation devices. As a bonus, our method can be inverted as follows: if the material properties are known with some confidence, our methodology can be used to obtain estimates of the underlying prestress of the substrate (with little additional information).

We believe that our work will be of great use in Medicine: tissues reveal their health through their "stiffness". Recently we have developed two novel indentation methods to test very soft materials like gels and developed the reverse analysis required for assessing the material properties of adhesive gels. We have also performed related tests on phantom materials that mimic very soft tissues like human organs (liver, brain etc), as well as artificial implants, grafts etc.

## 2. Materials and Method

Elastomers include many polymers and rubbers, as well as organic materials like human and animal tissues. Such materials are generally hard to test by conventional methods. The main idea is based on the indentation of surfaces of elastomer substrates by some form of rigid indentor (sphere, cone, pyramid) the measurement of indentation responses (applied force, surface deformation, contact area and time) and their interpretation to mechanical properties. Instrumented indentation of elastomers present theoretical and experimental challenges. Few analytic solutions are available [1, 2, 3], and the inverse indentation problem appears to have non-unique solution, even in the context of a particular strain energy function. We have developed measuring devices and procedures to obtain mechanical properties of such materials, circumventing the non-uniqueness problem. We have created prototypes for these devices and obtained two regional patents [4, 5]. Materials such as industrial rubber and in-house made gels have been tested with great success. The devices can be included at different stages of quality control in the manufacturing process. Other devices can be used for in situ measurements, depending on the end-user, increasing the safety level of operation of various components such as tires and belts. We are currently developing devices that can operate on a nano scale, in order to be able to capture properties of very small volumes such as tissues and very thin coatings.

### *2.1. Theory/calculation*

We examined axisymmetric contact problems in the absence of friction and adhesion. Starting with the work of Selvadurai and Spencer [6], we established a first order solution using an axisymmetric displacement potential $\psi(r,z)$ ( $r$ and $z$ are the radial and longitudinal cylindrical

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coordinates). The displacement potential must be harmonic $\left(\nabla^2\psi = 0\right)$ in order to satisfy the incompressibility condition. Far away from the contact region, the solution tends to the results of a concentrated point force $P$ acting normal to the surface of the contacted substrate, $\psi(r,z) = \dfrac{P}{\left(r^2 + z^2\right)^{1/2}}$. The overall force balance between the surface applied force through a rigid indenter and the reaction force applied by the far field is guaranteed by the local equilibrium equation of the contact stresses.

We then showed that, for elastomers with strain energy functions that are symmetric with respect to the principal strain invariants, the second order problem does not include the influence of the artificial body forces that appear in the regular second order analysis. We also used finite element analysis to verify and extend the above theoretical findings for a variety of well know energy density functions, such as the Mooney-Rivlin model, the Gent model and the Ogden model [7].

## 3. Results

The important result of the analytical and numerical analysis is that the force-displacement $\left(P-\delta\right)$ and the displacement-contact radius $\left(\delta - a\right)$ relations take the some forms of the ones obtained from the classic contact analysis for the incompressible linear elastic response [8]. In this formulation, the Poisson's ratio is $\nu = 1/2$ and the elastic modulus is $E = E_0$ (the tangent modulus at zero deformation). These approximations are valid for $\delta / a < 0.4$, that is for moderate strains and rotations. The particular results read as:

1. Flat punch:

$$P = \frac{8}{3} E_0 a \delta \tag{1}$$

2. Spherical punch of radius $R$:

$$P^2 = \frac{16}{27} E_0^2 R \delta^3 \tag{2a}$$

$$a^2 = \delta R \tag{2b}$$

3. Conical punch of angle $2\partial$:

$$P = \frac{8}{3\pi} \cot \partial E_0 \delta^2 \tag{3a}$$

$$a = \frac{2}{\pi} \frac{\delta}{\cot \partial} \tag{3b}$$

## 4. Discussion

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From the obtained results we can observe that the approximate relations that exist between the applied force $P$ and the vertical displacement $\delta$ of the indentor that comes into contact with the elastomer substrate involves only the initial elastic modulus $E_0$. Therefore, an instrumented indentation test can provide just this information. However, a general hyperelastic constitutive model may contain more than one constants, as for example the Mooney-Rivlin material that has a strain energy density function $W$ of the form

$$W = c_1 J_2 + c_2 \left( J_2 + 2J_1 \right) \tag{4}$$

where $c_1$ and $c_2$ are material constants and $J_1, J_2$ are the strain invariants of the principal stretches $\lambda_1, \lambda_2, \lambda_3$

$$J_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \tag{5a}$$

$$J_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_1^2 \lambda_3^2 - 2\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2\right) + 3 \tag{5b}$$

In this case $E_0 = 6(c_1 + c_2)$. This is a rigorous proof of non-uniqueness of the inverse problem: find the material constants from the $P - \delta$ response. Any reasonable combination of $c_1$ and $c_2$ can fit the indentation response, provided the sum $c_1 + c_2$ remains constant.

**Conclusions**

Elastic mechanical properties of elastomers can be found from indentation tests using analytic estimations. These properties can be correlated to the average molecular weight of the elastomers. The mechanical properties can be correlated with the state of the material, that is, existing residual stresses, fatigue level, solvent damage and radiation damage. Very small volumes of materials can be tested, making the procedure very useful to the micro and nano technologies. Toxic and hazardous substances can be readily checked with safety. Applications include testing of Human tissues, leading to complementary or preliminary diagnostic devices for diabetic and cancerous tissues – elastography.

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